

# A large neighbourhood based heuristic for two-echelon routing problems

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## Abstract

In this paper, we address two optimisation problems arising in the context of city logistics and two-level transportation systems. The two-echelon vehicle routing problem and the two-echelon location routing problem seek to produce vehicle itineraries to deliver goods to customers, with transits through intermediate facilities. To efficiently solve these problems, we propose a hybrid metaheuristic which combines enumerative local searches with destroy-and-repair principles, as well as some tailored operators to optimise the selections of intermediate facilities. We conduct extensive computational experiments to investigate the contribution of these operators to the search performance, and measure the performance of the method on both problem classes. The proposed algorithm finds the current best known solutions, or better ones, for 95% of the two-echelon vehicle routing problem benchmark instances. Overall, for both problems, it achieves high-quality solutions within short computing times. Finally, for future reference, we resolve inconsistencies between different versions of benchmark instances, document their differences, and provide them all online in a unified format.

## 1 Introduction

The traffic of vehicles is a major nuisance in densely populated areas. Trucks disturb peoples' well-being by emitting noise and air pollution. As the amount of goods in transit increases, a proper planning of road networks and facility locations becomes critical to mitigate congestion. To face these challenges, algorithmic tools have been developed to optimise city logistics at several levels: considering traffic regulation, itineraries and network design choices. Boosting the efficiency of goods transportation from suppliers to customers

presents important challenges for different planning horizons. On the operational level, efficient itineraries must be found for the available vehicles from day to day, e.g., reducing the travelled distance. On a tactical level, the overall fleet size, vehicle dimensions, capacities and characteristics are of interest. Larger trucks are more efficient in terms of cost per shipped quantity, whereas smaller vehicles are more desirable in city centres: they emit less noise, and only need smaller parking spots. Finally, the clever selection of locations for production sites, warehouses, and freight terminals is a typical strategic decision.

In this article, we consider the problem of jointly determining good routes to deliver goods to customers, and at which intermediate facilities a switch from larger trucks to smaller city freighters should happen. This problem is challenging, due to the combination of these two families of combinatorial decisions.

To address this problem, we propose a simple metaheuristic, which combines local and large neighbourhood search with the ruin and recreate principle. The method is conceptually simple and fast, exploiting a limited subset of neighbourhoods in combination with a simple strategy for closing and opening intermediate facilities. We conduct extensive computational experiments on two-echelon vehicle routing problem (2EVRP) and two-echelon location routing problem with single depot (2ELRPDS) instances to investigate the contribution of these operators, and measure the performance of the method on both problem classes. For the 2EVRP, this algorithm is able to reach or outperform 95% of the best known solutions. In general, for both problems, high-quality solutions are attained in short computing times. As such, this algorithm will serve as a good basis for future developments on more complex and realistic two-tiered delivery problems.

The paper is organised as follows. The problems are described in Section 2 and an overview of the related literature is given in Section 3. Mathematical formulations are presented in Section 4. Section 5 describes the proposed algorithm. Section 6 reviews current available benchmark instances and examines the performance of the proposed method. Section 7 concludes.

## 2 Problem description

Vehicle routing problems (VRPs) are a class of combinatorial optimisation problems, which aim to find good itineraries to service a number of customers with a fleet of vehicles. The 2EVRP is a variant of a VRP, which exploits the different advantages of small and large vehicles in an integrated delivery system. The goal is to design an efficient distribution chain, organised in two levels: trucks operate on the first level between a central depot and several selected intermediate distribution facilities, called satellites. The second level also includes the satellites – because both levels are interconnected there – as well as the end-customers. Small city freighters are operated between satellites and customers. The depot supplies sufficient quantities to satisfy all customer demands. The products are directly transferred from trucks to city freighters at satellite locations. These city freighters will perform the deliveries to the final customers. Any shipment or part of shipment has to transit through exactly one satellite, and the final delivery to the customer is done in one block. As such, split deliveries are not allowed for city freighters. The quantity (“demand”) of goods shipped to a satellite is not explicitly given, but evaluated as the sum of all customer demands served with city freighters originating from this satellite. Depending on the second level itineraries, split deliveries can occur on the first level since the total quantity needed at one satellite can exceed the capacity of one truck.

Finding good combined decisions for routing and intermediate facility openings is significantly more difficult than in well-studied settings such as the capacitated vehicle routing problem (CVRP). The special case of a 2EVRP with only one satellite can be seen as a VRP (Cuda et al., 2015; Perboli et al., 2011). The first level of the 2EVRP reduces to a CVRP with split deliveries. The structure of the second level is a multi-depot vehicle routing problem (MDVRP), where the depots correspond to the satellite locations (Jepsen et al., 2012). Those two levels have to be synchronised with each other. The 2EVRP is a generalisation of the classical VRP and is thus NP-hard.

Figure 1 shows different set-ups for goods distribution. The depot is represented by a triangle, satellites by squares, and customers by circles. Figures 1a and 1b show graphical representations of a split delivery vehicle routing problem (SDVRP) and a MDVRP respectively. Figures 1c–1e represent feasible solutions for the 2EVRP: with split deliveries occurring at one of the satellites, without split deliveries, and in Figure 1e a solution where only a subset of satellites is used.

The proposed algorithm has primarily been designed for the 2EVRP, and then tested on the 2ELRPDS, which includes additional tactical decisions. The basic structure of the 2ELRPDS is very similar to the 2EVRP.

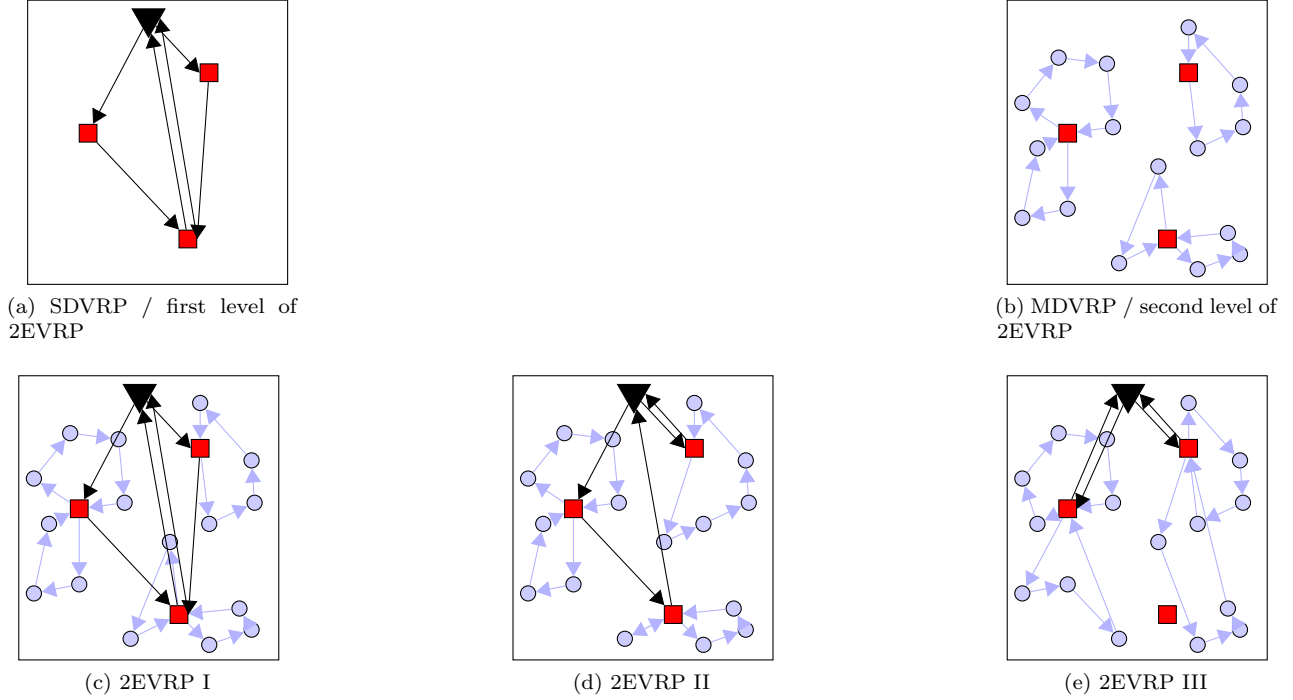


Figure 1: Subproblems related to the 2EVRP and different solutions depending on which intermediate facilities are used

The main difference is that it corresponds to a more tactical planning since only potential locations for depots or satellites are known and the use of any location results in opening costs. In contrast with the 2EVRP, the fleet size is unbounded, but fixed costs are counted for the use of each vehicle. The classical benchmark sets from the literature include different costs per mile for large first level trucks and small city freighters, unlike in 2EVRP benchmark instances, where mileage costs are identical for all vehicles. Finally, split deliveries are not allowed at both levels. We thus applied our algorithm to 2ELRPSD instances, where location decisions have to be taken at the secondary facilities. Following the notations of Boccia et al. (2011) we focus on  $3/T/\overline{T}$  problems.

### 3 Literature Review

Jacobsen and Madsen (1980) were amongst the first to introduce a two-echelon distribution optimisation problem. They proposed a three stage heuristic to solve the daily distribution of newspapers in Denmark, but no mathematical model was designed. Several possible transfer points were considered to transfer newspapers from one vehicle to another. The solution to this problem consists of three layers of decisions: the number and location of transfer points, the tours from the printing office and the tours from the transfer points to the retailers. An improved solution algorithm for the same problem can be found in Madsen (1983). Following the nomenclature of the authors and the classification in the recent survey on two-echelon routing problems by Cuda et al. (2015), the problem includes location decisions. Nevertheless, from today's point of view, it cannot be categorised as a two-echelon location routing problem (2ELRP) as there are no opening costs associated with the use of intermediate facilities and retailers can also be served directly from the printing office without using intermediate nodes. In addition, the problem cannot be categorised as a 2EVRP since first-level split deliveries are not allowed.

**Two-echelon vehicle routing problem** Crainic et al. (2004) used data from Rome to study an integrated urban freight management system. As large trucks cannot pass through the narrow streets in the city centre,

they used intermediate facilities to redistribute loads from large trucks to smaller vehicles. The city was divided into several commercial and external zones, and a mathematical location-allocation formulation was proposed and solved using a commercial solver. A comparison between solutions for delivering goods lead to the conclusion that intermediate facilities reduce the use of large trucks significantly, and more work is done by smaller city freighters.

Crainic et al. (2009) formulated a time dependent version of the problem, including time windows at the customers. To our knowledge, there are no test instances or solution approaches for this variant so far. Crainic et al. (2010) studied the impact of different two-tiered transportation set-ups on total cost. According to their results, the 2EVRP can yield better solutions than the VRP if the depot is not located within the customer area but externally. Perboli et al. (2011) introduced a flow-based mathematical formulation and generated three sets of instances for the 2EVRP with a maximum of 50 customers and four satellites, based on VRP instances. Their branch-and-cut approach is able to solve instances with up to 21 customers to optimality. Perboli and Tadei (2010) solved additional instances and reduced the optimality gap on others by means of new cutting rules.

Crainic et al. (2011) solved the 2EVRP with a multi-start heuristic. The method first assigns customers to satellites heuristically, and then solves the remaining VRPs with an exact method. In a perturbation step, the assignment of customers to satellites is changed, then the problem is solved again, until a number of iterations is reached.

Jepsen et al. (2012) presented a branch-and-cut method, solving 47 out of 93 test instances to optimality, 34 of them for the first time. The authors have been the first to consider a constraint on the number of vehicles *per satellite*, although it was already specified before in the existing data set. This additional constraint had not been taken into account by previous publications.

Hemmelmayer et al. (2012) developed a metaheuristic based on adaptive large neighbourhood search (ALNS) with a variety of twelve destroy and repair operators. This approach tends to privilege accuracy (high quality solutions) over simplicity and flexibility (Cordeau et al., 2002). The authors also introduced new larger test instances with up to 200 customers and five to ten satellites. Note that the results of ALNS on the problem instances with 50 customers cannot be compared with the proven optimal solutions by Jepsen et al. (2012), since the algorithm does not consider a limit on the number of vehicles per satellite, but rather a constraint on the total number of vehicles. For most problem instances, this algorithm found the current best known solution or improved it.

Santos et al. (2014) implemented a branch-and-cut-and-price algorithm, which relies on a reformulation of the problem to overcome symmetry issues. They also introduced several classes of valid inequalities. The algorithm performs well in comparison to other exact methods, and they reported solutions for instances with up to 50 customers.

Baldacci et al. (2013) presented a promising exact method to solve the 2EVRP. They decomposed the problem into a limited set of MDVRPs with side constraints. Detailed results and comparisons with previous publications were provided, as they considered both variants on the instances with 50 customers: with and without the constraint on vehicles per satellite. They also introduced a new set of instances with up to 100 customers.

Recently Zeng et al. (2014) published a greedy randomized adaptive search procedure, combined with a route-first cluster-second splitting algorithm and a variable neighbourhood descent. They also provide a mathematical formulation. They provide quite good results, although unfortunately their algorithm was tested only with instances comprising up to 50 customers.

**Two-echelon location routing problem** The capacitated 2ELRP is by far the most studied version of the 2ELRP. Many papers consider location decisions only at the second stage, either because the use of depots is an outcome of the first level routing optimisation, or they consider problems with only one single depot location which is known a priori (2ELRPD).

Laporte (1988) presented a general analysis of location routing problems and multi-layered problem variants. They compared several mathematical formulations and their computational performance. In a slightly different context, Laporte and Nobert (1988) formulated a vehicle flow model for the 2ELRP. The locations of the depots are assumed to be fixed and unchangeable, such that the location decisions only occur for the satellites. Following the notation by Boccia et al. (2011) they analysed  $3/R/\bar{R}$ ,  $3/R/\bar{T}$ ,  $3/T/\bar{R}$ , and  $3/T/\bar{T}$  problem settings.

Boccia et al. (2011) provided three mathematical formulations for the 2ELRP using one-, two-, and three indexed variables inspired from VRP and MDVRP formulations. A commercial solver was used to solve some instances generated by the authors, comparing two of the formulations in terms of speed and quality.

Nguyen et al. (2012a) introduced two new sets of instances for the 2ELRP. They implemented a GRASP with path relinking and a learning process and provided detailed results. In Nguyen et al. (2012b) the authors improved their findings on the same instances by using a multi-start iterated local search.

Contardo et al. (2012) proposed a branch-and-cut algorithm, which is based on a two-indexed vehicle flow formulation, as well as an ALNS heuristic. Both solution approaches were applied to one set of 2ELRP instances, and two sets of 2ELRP instances, outperforming previous heuristics.

Schwengerer et al. (2012) extended a variable neighbourhood search (VNS) solution approach for the location routing problem from Pirkwieser and Raidl (2010) and applied it to several instance sets, including the two aforementioned ones with a single depot.

For further details on both problem classes, we refer to the recent survey by Cuda et al. (2015). The previous literature review shows that, on the side of exact methods, the best approaches still cannot consistently solve (in practicable time) 2EVRP instances with more than 50 customers to optimality. On the side of heuristics, only few methods have been designed and tested to deal with instances with more than 50 delivery locations. To the best of our knowledge, only one heuristic has reported, to this date, computational results on larger 2EVRP instances (Hemmelmayer et al., 2012). Hence, there is a need for a more fine-grained study of solution methods for larger problems, as well as for simpler approaches able to efficiently deal with the two families of decisions related to routing and intermediate facilities selection. The proposed method has been designed to cope with these challenges. We developed a technique which performs very well on the classic benchmark instances and, in the meantime, uses fewer and simpler neighbourhood structures than previously published algorithms. During our research, we finally found inconsistencies regarding different benchmark instances used in previous papers. Some slightly different instances have also been referenced with the same name. Thus we collected the different versions and make them available online with unique names in a uniform file format, as described in Section 6.1.

## 4 Mathematical model

Different mathematical formulations have been proposed for the 2EVRP (Perboli et al., 2011; Jepsen et al., 2012; Baldacci et al., 2013; Santos et al., 2013, 2014) and for the 2ELRP (Boccia et al., 2011; Contardo et al., 2012). In this section, we display compact formulations based on the model of Cuda et al. (2015).

The 2EVRP can be defined on a weighted undirected graph  $G = (N, E)$ , where the set of vertices  $N$  consists of the depot  $\{0\}$ , the set of possible satellite locations  $S = \{1, \dots, |S|\}$  and the set of customers  $C = \{|S| + 1, \dots, |S| + |C|\}$ . The set of edges  $E$  is divided into two subsets, representing the first and second echelon respectively. Set  $E^1 = \{(i, j) : i < j, i, j \in \{0\} \cup S\}$  represents the edges which can be traversed by first-level vehicles: those connecting the depot to the satellites, and those interconnecting satellites with each other. The set of edges  $E^2 = \{(i, j) : i < j, i, j \in S \cup C, (i, j) \notin S \times S\}$  is used for the second level, and corresponds to possible trips between satellite and customers or pairs of customers.

A fleet of  $v^1$  homogeneous trucks with capacity  $Q^1$  is located at the depot. A total of  $v^2$  homogeneous city freighters are available, each with a given capacity of  $Q^2$ . They can be located at any satellite  $s \in S$ . Still, the number of city freighters at one satellite is limited to  $v_s^2$ .

The set  $R^1$  contains all possible routes starting from the depot and delivering a given sequence of customers, then returning to the depot again. Similarly each route  $r$  in the set of secondary routes  $R^2$  starts at a satellite  $s \in S$ , visits one or several customers in  $C$ , and returns again to satellite  $s$ . Each customer  $c \in C$  has a demand of  $d_c$  units. Each unit of freight shipped through a satellite induces a handling cost  $h_s$ .

Given a secondary route  $r \in R^2$  and a customer  $c \in C$ , the parameter  $\beta_{rc} \in \{0, 1\}$  is equal to 1 if and only if customer  $c$  is visited in route  $r$ , and 0 otherwise. Let  $d_r = \sum_{c \in C: c \in r} d_c \leq Q^2$  denote the total demand of customers visited in route  $r$ , and  $p_r$  represents the cost of each route  $r \in R^1 \cup R^2$ . The binary variables  $x_r \in \{0, 1\}$  with  $r \in R^1 \cup R^2$  take the value 1 if and only if route  $r$  is in the solution. Finally, each decision variable  $q_{rs} \geq 0$  with  $r \in R^1$ ,  $s \in S \cap r$ , gives the load on the truck on route  $r$  that has to be delivered to satellite  $s$ .

$$\min \sum_{r \in R^1 \cup R^2} p_r x_r + \sum_{s \in S} h_s \sum_{r \in R^1} q_{rs} \quad (1)$$

s.t.

$$\sum_{r \in R^1} x_r \leq v^1 \quad (2)$$

$$\sum_{r \in R^2} x_r \leq v^2 \quad (3)$$

$$\sum_{r \in R^2: s \in r} x_r \leq v_s^2 \quad s \in S \quad (4)$$

$$\sum_{s \in S \cap r} q_{rs} \leq Q^1 x_r \quad r \in R^1 \quad (5)$$

$$\sum_{r \in R^1} q_{rs} = \sum_{r \in R^2: s \in r} d_r x_r \quad s \in S \quad (6)$$

$$\sum_{r \in R^2} \beta_{rc} x_r = 1 \quad c \in C \quad (7)$$

$$x_r \in \{0, 1\} \quad r \in R^1 \cup R^2 \quad (8)$$

$$q_{rs} \geq 0 \quad r \in R^1, s \in S \cap r \quad (9)$$

The objective function (1) sums up routing costs for all routes on both levels and handling costs per unit moved through each satellite. Constraints (2) and (3) set the number of available vehicles for trucks and city freighters, respectively. The number of city freighters per satellite is constrained by Constraint (4). Constraints (5) ensure that the maximum capacity of the trucks is not exceeded. Constraints (6) link the quantities of goods between the first and the second level. They guarantee that the incoming goods equal the outgoing goods at the satellites. As there are no split deliveries allowed on the second level, Constraints (7) ensure that each customer is visited exactly once. The domains of the decision variables are defined by Constraints (8) and (9).

The mathematical model for the 2ELRPSD is similar to the previous model, but needs some adjustments. Each satellite  $s \in S$  has a given opening cost  $f_s$  and a capacity of  $k_s$  units of freight. There is an unbounded number of vehicles available at both levels. Therefore, Constraints (2) to (4) are not needed. The routing of a vehicle incurs fixed costs of  $f^1$  for each truck, and  $f^2$  for each used city freighter. Each binary parameter  $\alpha_{rs}$  is equal to 1 if and only if satellite  $s$  is visited on route  $r$ , and 0 otherwise. If satellite  $s$  is opened in the solution, binary variable  $y_s$  takes value 1, and 0 otherwise.

Using Constraints (5) to (9), the objective function needs to be changed to  $\min \sum_{r \in R^1} (f^1 + p_r) x_r + \sum_{r \in R^2} (f^2 + p_r) x_r + \sum_{s \in S} f_s y_s$  to consider fixed and mileage-based vehicle costs for both levels separately, as well as opening costs for satellites. The capacity limit at the satellites is imposed by  $\sum_{r \in R^1} q_{rs} \leq k_s y_s$  for all  $s \in S$ . If a satellite  $s$  has been selected to be open, then the delivery by exactly one truck is guaranteed by Constraints  $\sum_{r \in R^1} \alpha_{rs} x_r = y_s$  for all  $s \in S$ .

## 5 Solution method

The proposed metaheuristic follows the basic structure of a large neighbourhood search (LNS), which was first introduced by Shaw (1998). An initial feasible solution is iteratively destroyed and repaired in order to gradually improve the solution. Such a ruin and recreate approach (Schrimpf et al., 2000) has been successfully applied to multiple variants of vehicle routing problems in the past (see, e.g., Pisinger and Ropke 2010). The destruction of parts of a previous solution (ruin) gives freedom to create a new and better solution (recreate). Algorithm 1 shows the basic structure of the proposed method.

At each iteration of the proposed method, 1) a partial solution destruction is performed on the routes of the second level; 2) then the second level is repaired and improved by means of local search, and finally 3) the

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**Algorithm 1: LNS-2E**

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```
1  $S^{best} \leftarrow S \leftarrow localSearch(repair(instance))$  /* initial solution */
2  $g \leftarrow 0$ 
3 repeat
4   for  $i \leftarrow 0$  to  $i_{max}$  do
5      $S^{temp} \leftarrow localSearch(repair(destroy(S, g)))$ 
6     if Satellite was opened/closed during previous destroy phase then
7        $g \leftarrow 0$  /* reset grace period */
8     if  $cost(S^{temp}) < cost(S)$  then
9        $S \leftarrow S^{temp}$  /* accept better solution */
10       $i \leftarrow 0$  /* reset re-start period */
11     $g \leftarrow g + 1$ 
12  if  $cost(S) < cost(S^{best})$  then
13     $S^{best} \leftarrow S$  /* store best solution */
14  else
15     $S \leftarrow localSearch(repair(instance))$  /* re-start: new solution */
16 until  $time > time_{max}$ 
17 return  $S^{best}$ 
```

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first level is reconstructed with a simple heuristic. As such, the first level is constructed from scratch in every iteration, but since the number of nodes in the first-level sub-problem is relatively small, this simple heuristic already finds an optimal or near-optimal solution.

Each of the *destroy* phases performs all the destroy operators sequentially as they are described in Section 5.1. One single *repair* mechanism is used for solution reconstruction and also to obtain an initial solution (Line 1 and 5). This procedure is described in Section 5.2. Afterwards, as needed quantities at the satellites are known, the first level is reconstructed as described in Section 5.3.

The choices of intermediate facilities may change as a consequence of the repair operator, or through dedicated destroy operators which temporarily close or re-open some possible locations for intermediate facilities. If a change in open or closed satellites has recently taken place, the status of another satellite will not be changed (Line 5) for a number of iterations that we call *grace period* ( $g$  is reset to 0 in Line 7).

We then put emphasis on a strong local search phase, exploiting well-known procedures like 2-opt (Croes, 1958), 2-opt\*, or simple relocate and swap moves. *Relocate* shifts a node before or after one of the closest neighbours, if costs are improved. *Swap* explores the exchange of one node with one of the neighbour nodes, as well as exchanging one node with two successive neighbour nodes. This local search phase is applied after the destroy and repair operators. In order to reduce the complexity, moves are only attempted between close customers, as done in the granular search by Toth and Vigo (2003). Further information on these moves can be found in the survey by Vidal et al. (2013).

If a better solution is obtained, it is accepted as the new incumbent solution (Line 9). If no improvement can be found for a large number of iterations, then the algorithm will restart from a new initial solution, even if the objective value is worse (Line 15).

In general, our algorithm requires less local and large neighbourhood operators than the ALNS proposed by Hemmelmayr et al. (2012). The destruction operator parameters are also selected randomly, since the method performed equally well, during our computational experiments, without need for a more advanced adaptive scoring system. In the following, we describe the sets of destroy and repair operators, as well as the management of the decisions related to the first level.

## 5.1 Destroy operators

Our algorithm relies on different destroy operators which are all invoked at each iteration in sequential order. They are applied only to the second level. All of them, except the *open all satellites* neighbourhood, select nodes which are removed from the current solution.

The first four destroy operators are used at each iteration. The last two ones, which change the status of a satellite to *closed* or *open* again, are only invoked if  $g$  in Line 5 of Algorithm 1 has exceeded the grace period  $g_{max}$  (i.e. no change in open/closed satellites has taken place recently). The destroy operators are now described, in their order of use. When applicable, all random samples are uniformly distributed within their given interval.

**Related node removal** A seed customer is randomly chosen. A random number of its Euclidean closest customers as well as the seed customer are removed from the current solution and added to the list of nodes to re-insert. This operator receives a parameter  $p_1$ , which denotes the maximum percentage of nodes to remove. At most  $\lceil p_1 \cdot |C| \rceil$  nodes are removed, with  $|C|$  being the overall number of customers.

**Biased node removal** First, the removal cost of each customer is computed: the savings associated to a removal of node  $j$ , located between  $i$  and  $k$ , is given by  $\delta_j = c_{ik} - c_{ij} - c_{jk}$ , where  $c_{ij}$  denotes the travel cost from node  $i$  to node  $j$ . The probability of selection of a node for removal is then linearly correlated with the delta evaluation value. The higher the gain after removal, the more likely it will be selected and removed. In every destroy phase, a random percentage of customers from the interval  $[0, p_2]$  is removed.

**Random route removal** Randomly selects routes and removes all containing customers, adding them to the list of nodes to re-insert. This operator randomly selects a number of routes in the interval  $[0, \lceil p_3 \cdot \sum_{c \in C} \frac{d_c}{Q^2} \rceil]$ .

**Remove single node routes** This operator removes all routes which contain only one single customer. In the case of the 2EVRP there is a limited number of overall vehicles available, and thus removing the short routes allows to use a vehicle originating from a different satellite in the next repair phase. This operator is used with a probability of  $\hat{p}_4$ .

The last two destroy operators can be used at most at each  $g_{max}$  iterations. During this grace period after satellite selection has been actively altered, none of these two operators will be executed.

**Close satellite** Chooses a random satellite. If the satellite can be closed and the remaining open ones still can provide sufficient capacity for a feasible solution, the chosen satellite is closed temporarily. All the customers, which are assigned to it, are removed and added to the list of nodes to re-insert. The satellite stays closed until it is opened again in a later phase. This operation is chosen with a probability of  $\hat{p}_5$ , given variable  $g$  has already exceeded the grace period. If this operator has been executed,  $g$  is reset to 0.

**Open all satellites** This neighbourhood makes all previously closed satellites available again. It comes into effect with a probability  $\frac{\hat{p}_5}{|S|}$ , and thus it depends on the same parameter as the *close satellite* operator and the number of satellites. This operator can only be executed if  $g > g_{max}$ , i.e. outside the grace period. Its execution resets  $g$  to 0.

## 5.2 Repair operator, randomisation and initial solution

At each repair phase, the insertion of the nodes is done in random order. This repair mechanism can sometimes fail, if one customer remains with a higher demand than the largest free capacity available on any vehicle. In this exceptional case, the repair process is restarted, and the nodes are inserted by decreasing demands to preserve feasibility.

Repair is achieved with a simplified cheapest insertion heuristic. All nodes are sequentially inserted at their cheapest possible position in the solution. The main difference with the classic cheapest insertion heuristic is



that the method does not aim to insert the node with the lowest increase in total costs, but just takes the next candidate from the list and inserts it, in order to reduce complexity and enhance solution diversity. It is a simple and greedy heuristic.

Both the initial solution and every partial solution are always repaired by the same operator. The initial solution can be seen as a “completely destroyed” solution.

After the second level has been repaired, the local search procedure is performed: 2-opt on each of the routes, 2-opt\* on all routes originating at the same satellite. The algorithm then tries to relocate single nodes, swap one node with another and to swap two nodes with one other, within a limited neighbourhood of the  $\tau$  closest nodes, again accepting only improvements. This procedure stops when no improving move exists in the entire neighbourhood. After this procedure, the delivery quantity of each of the satellites is known, and the first level can be constructed using the same insertion heuristic as for the second level and performing local search.

### 5.3 Reconstruction of the first level

For the 2EVRP, it is essential to allow satellites to be delivered by several trucks. In particular, if the demanded quantity at a satellite is larger than a full truckload and no other satellite is available, then there would be no feasible solution. To reconstruct a first level solution, we propose a very simple preprocessing step. Any satellite with a demand larger than a full truckload is virtually duplicated into nodes with demands equal to a truckload, until the remaining demand is smaller than  $Q^1$ . The same insertion procedure as the repair operator is used to generate a first level solution. This creates back-and-forth trips to the virtual nodes with demands equal to a full truckload, and completes the solution analogously for the remaining nodes.

Usually there are few nodes associated with the SDVRP on the first level. The largest benchmark instances from literature so far contain only ten satellites at most. This very simple policy enabled to find nearly-optimal first level solutions for most considered instances with limited computational effort. Finally, note that in the considered 2ELRPSD instances, the capacity of a satellite is never larger than the trucks’ capacity. Therefore, split deliveries are not generated during reconstruction.

## 6 Computational Experiments

This section describes the currently available sets of instances for the 2EVRP (in Section 6.1) and the used instances for the 2ELRPSD (Section 6.2) and attempts to resolve some inconsistencies. The calibration of the method is described in Section 6.3. The computational results and the comparisons with other state-of-the-art algorithms are discussed in Sections 6.4 and 6.5. Finally, Section 6.6 analyses the sensitivity of the method with respect to several key parameters and design choices.

### 6.1 Benchmark Instances for the two-echelon vehicle routing problem

When looking at the literature, it may appear that there are six unique sets of benchmark instances. However, due to inconsistencies with respect to constraints, nomenclature or locations, we identified in fact several different subsets. In what follows, we explain the differences and provide high quality solutions for them. We consider five different sets of benchmark instances from literature. Sets 2 and 3 were proposed by Perboli et al. (2011) and have been generated based on the instances for the CVRP by Christofides and Eilon. Different customers were chosen and converted into satellites. They also proposed the small Set 1 instances, with just twelve customers and two satellites, which we did not consider. Set 4 was proposed by Crainic et al. (2010); all of them were downloaded from OR-Library (Beasley, 2014).

The instance Sets 2 to 5 as used in Hemmelmayr et al. (2012) were also communicated to us by email (Hemmelmayr, 2013). We noticed a few key differences with the ones available from Beasley (2014).

Set 6 instances were provided from the authors (Baldacci, 2013).

All distances are Euclidean, and computed with double precision. Note that handling costs are set to 0 for all sets except 6b. We will now explain the characteristics of these instance sets in detail, and propose unique names for the sets to overcome existing inconsistencies:

**Set 2** There are two different versions in circulation: Please note that the instances with 50 customers in the OR-Library contain a mistake<sup>1</sup>. This can be resolved by exchanging  $Q^1$  and  $Q^2$  capacity values, which is also the way we treated them, like previous authors did.

The names of the instances downloaded from Beasley (2014) and used by Hemmelmayr et al. (2012) were the same, but the instances with 50 customers included different locations for the satellites. For future reference we provide both versions, and we rename the instances with less than 50 customers to Set 2a, the Hemmelmayr (2013) version of 50 customer Set 2 instance files to Set 2b, and the OR-Library version will be called Set 2c. Table 3 shows the characteristics of all Set 2 instances.

Instance names used by Baldacci et al. (2013) have the satellite numbers incremented by one. Apart from that they are identical with what we received from Hemmelmayr (2013). For example Set 2a instance named E-n51-k5-s2-17 (Satellites 2 and 17) corresponds to E-n51-k5-s3-18 in the result tables of Baldacci et al. (2013).

We provide both versions (OR-Library with corrected capacities as well as the ones received by Hemmelmayr) with distinguishable names at <https://www.univie.ac.at/prolog/research/TwoEVRP>.

**Set 3** There are also two different versions of the Set 3 instances in circulation. We collected and solved all of them and distinguished between different versions and identified inconsistencies. Again, the sources Beasley (2014) and Hemmelmayr (2013) were identical for instances with 21 and 32 customers, but different for instances with 50 customers. In the case of Set 3, the filenames for the different instances were also different, so there is no need to introduce new distinguishable names.

The only difference between Set 3 instances with 50 customers from the two sources is the location of the depot. The locations of satellites and customers, as well as the vehicles and demands are identical. All Set 3 instances from Hemmelmayr (2013) place the depot at coordinates (0,0), whereas the files of Beasley (2014) have the depot located at (30,40). Table 13 shows which instances correspond to each other, apart from satellite location.

Please also note that like in Set 2, the Set 3 instances with 50 customers from the OR Library also have the capacities of the two vehicle types interchanged (see Footnote 1). This has been corrected in the files which we provide online.

For easier referencing, we also divide the instances in three parts. Set 3a includes all instances with less than 50 customers, Set 3b the larger instances which have been used by Hemmelmayr et al. (2012), and Set 3c the larger instances as they are available at the OR-Library, and have been used by Baldacci et al. (2013), among others.

**Set 4** These instances have been treated differently in literature, either with a limit on the number of second level vehicles allowed *per satellite* or only considering a total number of vehicles, with no limitations on the distribution amongst satellites. As proposed in Baldacci et al. (2013), we solved both versions and follow their nomenclature: Set 4a with the limit per satellite, and Set 4b when the constraint of vehicles per satellite is relaxed.

**Set 5** This set of instances has been proposed by Hemmelmayr et al. (2012). To the best of our knowledge they were the only ones to report solutions on all instances of that set. Baldacci et al. (2013) were able to find solutions on the small instances with only five satellites.

**Set 6** To the best of our knowledge, solutions on these instances have only been reported in Baldacci et al. (2013). Set 6 includes two subsets: Set 6a, with  $h_s = 0$ , and Set 6b, which considers different handling costs per freight unit at each of the satellites.

Table 1 displays an overview of the characteristics of the individual sets. It lists the number of instances in the according set and subset with number of customers (C), satellites (S), trucks (T), city freighters (CF) and available city freighters per satellite  $v_s^2$ . Column hC shows if the handling costs are non-zero. The source of the instance sets is also provided (Hemmelmayr, 2013; Beasley, 2014; Baldacci, 2013).

<sup>1</sup>First level vehicles have a capacity of  $Q^1 = 160$  units, and second level vehicles, which by design are supposed to be smaller than level 1 trucks, have a capacity of  $Q^2 = 400$  units. For instances with two satellites for example, there are 3 trucks available. They can ship a maximum of  $3 * 160 = 480$  units. Overall customers' demand  $\sum_{c \in C} d_c$  is larger than 480 units, so there is per se no feasible solution for those instances.

Table 1: Characteristics and Sources of Instance Sets

Set	Subset	Inst.	C	S	T	CF	hC	$v_s^2$	HCC	OR-Library	Baldacci
2	a	6	21	2	3	4	-	-	•	•	
		6	32	2	3	4	-	-	•	•	
	b	6	50	2	3	5	-	-	•		
		3	50	4	4	5	-	-	•		
	c	6	50	2	3	5	-	-		•	
		3	50	4	4	5	-	-		•	
3	a	6	21	2	3	4	-	-	•	•	
		6	32	2	3	4	-	-	•	•	
	b	6	50	2	3	5	-	-	•		
		6	50	2	3	5	-	-		•	
	c	6	50	2	3	5	-	-			•
		6	50	2	3	5	-	-			•
4	a	18	50	2	3	6	4	-		•	
		18	50	3	3	6	3	-		•	
		18	50	5	3	6	2	-		•	
	b	18	50	2	3	6	-	-	•		
		18	50	3	3	6	-	-	•		
		18	50	5	3	6	-	-	•		
5		6	100	5	5	[15,32]	-	-	•		
		6	200	10	5	[17,35]	-	-	•		
		6	200	10	5	[30,63]	-	-	•		
6	a	9	50	[4,6]	2	50	-	-			•
		9	75	[4,6]	3	75	-	-			•
		9	100	[4,6]	4	100	-	-			•
	b	9	50	[4,6]	2	50	•	-			•
		9	75	[4,6]	3	75	•	-			•
		9	100	[4,6]	4	100	•	-			•

## 6.2 Benchmark Instances for the two-echelon location routing problem

The proposed algorithm was originally designed for the 2EVRP, nevertheless we also tested it on benchmark instances for the 2ELRPD. Two sets, called “Nguyen” and “Prodhon” are available at [http://prodhon.free.fr/Instances/instances0\\_us.htm](http://prodhon.free.fr/Instances/instances0_us.htm). They present some small errors or unclear descriptions, which are documented in Appendix 8.3.

## 6.3 Parameters

The parameters of the proposed method have been calibrated using meta-calibration: the problem of finding good parameters is assimilated to a black-box optimisation problem, in which the method parameters are the decision variables, and the objective function is simulated by running the method on a set of training instances, containing five randomly selected instances, for each set. To perform a fast optimisation we rely on the covariance matrix adaptation evolution strategy (CMA-ES) by Hansen (2006). The source code (in Java) is available at [https://www.lri.fr/~hansen/cmaes\\_inmatlab.html](https://www.lri.fr/~hansen/cmaes_inmatlab.html).

The performance of our algorithm is rather insensitive to changes in parameters for the small instances, but the rules for closing and opening satellites have to be adjusted to the number of overall available satellites. Our calibration experiments have been conducted for each instance set, independently, and then we searched for one compromise setting for the parameters that yields satisfying results for all different benchmark instances. The calibration results are displayed in Table 2, as well as the average value, standard deviation, and the compromise value which was used for the runs reported in Section 6.4.

The size of the limited neighbourhood for the local search relocate and swap moves was also determined by CMA-ES. This parameter always converged to  $\tau = 25$  already in early stages of the tuning process, and thus relocate and swap moves are attempted only for nodes within the radius including the 25 Euclidean closest nodes.

Table 2: Parameter values obtained by meta-calibration

	2EVRP							2ELRPSD		Mean	Std. Dev.	compromise
	Set 2	Set 3	Set 4a	Set 4b	Set 5	Set 6a	Set 6b	Nguyen	Prodhon			
$p_1$	0.35	0.39	0.32	0.31	0.29	0.33	0.26	0.14	0.34	0.30	0.07	0.20
$p_2$	0.18	0.20	0.12	0.07	0.80	0.52	0.78	0.50	0.19	0.37	0.27	0.35
$p_3$	0.09	0.07	0.14	0.16	0.14	0.21	0.19	0.28	0.17	0.16	0.06	0.25
$\hat{p}_4$	0.33	0.73	0.19	0.09	0.32	0.21	0.41	0.37	0.57	0.36	0.18	0.50
$\hat{p}_5$	0.06	0.01	0.29	0.24	0.14	0.28	0.26	0.21	0.20	0.19	0.09	0.20

## 6.4 Computational Results

As done in previous literature, we performed five independent runs on each of the 2EVRP benchmark instances and 20 runs on the 2ELRPSD instances. The code is written in Java with JDK 1.7.0.51 and tested on a Intel E5-2670v2 CPU at 2.5 GHz with 3 GB RAM. The code was executed single threaded on one core. We compare the performance of our method on the 2EVRP instances with the hybrid GRASP +VND by Zeng et al. (2014) and the ALNS by Hemmelmayr et al. (2012), when applicable; as well as the currently best known solutions for each instance from the literature. We also show the results of the algorithm on the 2ELRP with single depot and compare with the VNS of Schwengerer et al. (2012). We describe the data of the following tables in general and discuss results in detail on each of the instance sets separately.

Tables 3 to 11 show the characteristics and detailed results for each instance. The columns  $C$ ,  $S$ ,  $T$  and  $CF$  display the main characteristics of the instance, where  $C$  is the number of customers,  $S$  is the number of satellites,  $T$  and  $CF$  the number of available trucks and city freighters, respectively. The last two columns are not applicable for Tables 10 to 11, as they correspond to 2ELRPSD instances with unbounded fleet size.

The next columns display the results of the proposed method (LNS-2E), and methods by Hemmelmayr et al. (2012) (HCC), Zeng et al. (2014) (ZXXS) for the 2EVRP when applicable, and Schwengerer et al. (2012) (SPR) for the 2ELRPSD. The average objective value of five runs is given in column *Avg. 5*. Column *Best 5* shows the best solution found within these five runs, and *Best* gives the best objective value found during all experiments, including parameter calibration. Following the work of Schwengerer et al. (2012), we also used average and best of 20 for the 2ELRPSD for better comparison.

Column  $t$  reports the average overall runtime of the algorithms in seconds, and  $t^*$  the average time when the best solution was found. For easier comparison we chose a simple time limit for termination of our algorithm: 60 seconds for instances with up to 50 customers, and 900 seconds for larger ones. Our time measure corresponds to the wall-clock time of the whole execution of the program, including input and output, computation of the distance matrix, and other pre-processing tasks. Hemmelmayr et al. (2012) and Schwengerer et al. (2012) report CPU times (which may be slightly smaller than wall clock times). Zeng et al. (2014) only report the time when the best solution was found, but no overall runtime of the algorithm.

*BKS* refers to the best known solution of that instance. Best known solutions are highlighted in boldface when found by the algorithm, and new BKS are also underlined. We highlight an instance with an asterisk after BKS if the best known solution of the instance is known to be optimal from previous literature.

Tables 3 and 4 provide detailed results on the instances of Set 2 and Set 3. HCC, ZXXS and our algorithm find the best known solutions at every run. The solutions have been proven to be optimal for all the instances except Set 2c and Set 3b. To the best of our knowledge, we are the first ones to report solutions on the 2c instances obtained from Beasley (2014). For Set 3c, the optimal objective values are derived from Baldacci et al. (2013) and Jepsen et al. (2012), but no results from HCC or ZXXS are available. Summarising Tables 3 and 4, we can conclude that Sets 2 and 3 are easy in the sense that all runs of all algorithms always found the optimal or best known solution.

The instances of Set 4 have been addressed in various ways in the literature. Jepsen et al. (2012) considered a limit on the number of city freighters available at each satellite, HCC and ZXXS did not impose this limit, and instead considered the limit on the total number of city freighters only. Baldacci et al. (2013) addressed both variants of the instances to compare their results to both previous results, introducing a new nomenclature: Set 4a for the instances including the limit of vehicles per satellite, and Set 4b when this limit is relaxed.

Tables 5 and 6 display the results on Set 4a and 4b instances. 102 out of the 108 instances have been solved to optimality by Baldacci et al. (2013). Nevertheless we observed small differences of objective values with

our solutions (up to a 0.006% difference). This could be explained by a different rounding convention (we use double precision), or by the small optimality gap of Cplex. As a consequence, bold fonts were used for BKS within 0.006% precision. In all these cases the underlying solution is identical, just the objective value is marginally different. We can see from Tables 5 and 6 that in all cases our best solution corresponds to the optimal or best known solution. Only in 3 and 2 instances of Set 4a and 4b, respectively, some of the runs gave slightly worse solutions. On average, in instance Set 4b our results are slightly better than those of the other heuristics.

To the best of our knowledge, Hemmelmayr et al. (2012) were the only authors who published results on the large Set 5 instances with 10 satellites to this date. Baldacci et al. (2013) report solutions on the small Set 5 instances (100 customers/5 satellites), improving three out of six instances to optimality. The algorithm of Hemmelmayr et al. (2012) was evaluated with a limit of 500 iterations. We compare our results in Table 7 and were able to improve the best known solutions on 9 of the 18 instances, depicted with an underlined BKS value. Known optimal solutions are retrieved at least once within the five performed test runs.

Tables 8 and 9 report the results for the instances of Set 6a and b. From the 54 instances, all except 13 solutions have been proven to be optimal, and on nine of those remaining LNS-2E was able to find better solutions. Best solutions were found typically after less than three minutes.

Table 3: Results for Set 2 Instances

Instance	C	S	T	CF	HCC			ZXXS			LNS-2E					BKS
					Avg. 5	t(s)	t*(s)	Avg. 5	t*(s)	Avg. 5	Best 5	Best	t(s)	t*(s)		
<b>Set 2a<sup>1,2</sup></b>																
E-n22-k4-s6-17	21	2	3	4	<b>417.07</b>	37	0	<b>417.07</b>	0	<b>417.07</b>	<b>417.07</b>	<b>417.07</b>	<b>417.07</b>	60	1	417.07*
E-n22-k4-s8-14	21	2	3	4	<b>384.96</b>	34	0	<b>384.96</b>	0	<b>384.96</b>	<b>384.96</b>	<b>384.96</b>	<b>384.96</b>	60	1	384.96*
E-n22-k4-s9-19	21	2	3	4	<b>470.60</b>	35	0	<b>470.60</b>	0	<b>470.60</b>	<b>470.60</b>	<b>470.60</b>	<b>470.60</b>	60	1	470.60*
E-n22-k4-s10-14	21	2	3	4	<b>371.50</b>	37	0	<b>371.50</b>	0	<b>371.50</b>	<b>371.50</b>	<b>371.50</b>	<b>371.50</b>	60	2	371.50*
E-n22-k4-s11-12	21	2	3	4	<b>427.22</b>	31	0	<b>427.22</b>	0	<b>427.22</b>	<b>427.22</b>	<b>427.22</b>	<b>427.22</b>	60	2	427.22*
E-n22-k4-s12-16	21	2	3	4	<b>392.78</b>	36	0	<b>392.78</b>	0	<b>392.78</b>	<b>392.78</b>	<b>392.78</b>	<b>392.78</b>	60	1	392.78*
E-n33-k4-s14-22	32	2	3	4	<b>779.05</b>	85	0	<b>730.16</b>	0	<b>779.05</b>	<b>779.05</b>	<b>779.05</b>	<b>779.05</b>	60	1	779.05*
E-n33-k4-s1-9	32	2	3	4	<b>730.16</b>	74	0	<b>714.63</b>	0	<b>730.16</b>	<b>730.16</b>	<b>730.16</b>	<b>730.16</b>	60	1	730.16*
E-n33-k4-s2-13	32	2	3	4	<b>714.63</b>	64	0	<b>707.48</b>	0	<b>714.63</b>	<b>714.63</b>	<b>714.63</b>	<b>714.63</b>	60	1	714.63*
E-n33-k4-s3-17	32	2	3	4	<b>707.48</b>	58	0	<b>778.74</b>	1	<b>707.48</b>	<b>707.48</b>	<b>707.48</b>	<b>707.48</b>	60	1	707.48*
E-n33-k4-s4-5	32	2	3	4	<b>778.74</b>	77	3	<b>756.85</b>	0	<b>778.74</b>	<b>778.74</b>	<b>778.74</b>	<b>778.74</b>	60	1	778.74*
E-n33-k4-s7-25	32	2	3	4	<b>756.85</b>	53	0	<b>779.05</b>	0	<b>756.85</b>	<b>756.85</b>	<b>756.85</b>	<b>756.85</b>	60	1	756.85*
Avg.					<b>577.59</b>	51	0	<b>577.59</b>	0	<b>577.59</b>	<b>577.59</b>	<b>577.59</b>	<b>577.59</b>	60	1	577.59
<b>Set 2b<sup>1</sup></b>																
E-n51-k5-s11-19	50	2	3	5	<b>581.64</b>	182	6	<b>597.49</b>	1	<b>581.64</b>	<b>581.64</b>	<b>581.64</b>	<b>581.64</b>	60	1	581.64*
E-n51-k5-s11-19-27-47	50	4	4	5	<b>527.63</b>	147	1	<b>530.76</b>	0	<b>527.63</b>	<b>527.63</b>	<b>527.63</b>	<b>527.63</b>	60	4	527.63*
E-n51-k5-s2-17	50	2	3	5	<b>597.49</b>	100	7	<b>554.81</b>	1	<b>597.49</b>	<b>597.49</b>	<b>597.49</b>	<b>597.49</b>	60	3	597.49*
E-n51-k5-s2-4-17-46	50	4	4	5	<b>530.76</b>	154	1	<b>581.64</b>	4	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	60	3	530.76*
E-n51-k5-s27-47	50	2	3	5	<b>538.22</b>	136	1	<b>538.22</b>	1	<b>538.22</b>	<b>538.22</b>	<b>538.22</b>	<b>538.22</b>	60	1	538.22*
E-n51-k5-s32-37	50	2	3	5	<b>552.28</b>	141	1	<b>552.28</b>	1	<b>552.28</b>	<b>552.28</b>	<b>552.28</b>	<b>552.28</b>	60	2	552.28*
E-n51-k5-s4-46	50	2	3	5	<b>530.76</b>	173	0	<b>530.76</b>	1	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	60	3	530.76*
E-n51-k5-s6-12	50	2	3	5	<b>554.81</b>	149	2	<b>531.92</b>	1	<b>554.81</b>	<b>554.81</b>	<b>554.81</b>	<b>554.81</b>	60	4	554.81*
E-n51-k5-s6-12-32-37	50	4	4	5	<b>531.92</b>	150	0	<b>527.63</b>	1	<b>531.92</b>	<b>531.92</b>	<b>531.92</b>	<b>531.92</b>	60	2	531.92*
Avg.					<b>549.50</b>	148	2	<b>549.50</b>	1	<b>549.50</b>	<b>549.50</b>	<b>549.50</b>	<b>549.50</b>	60	2	549.50
<b>Set 2c<sup>2</sup></b>																
E-n51-k5-s11-19	50	2	3	5						<b>617.42</b>	<b>617.42</b>	<b>617.42</b>	<b>617.42</b>	60	3	617.42
E-n51-k5-s11-19-27-47	50	4	4	5						<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	60	1	530.76
E-n51-k5-s2-17	50	2	3	5						<b>601.39</b>	<b>601.39</b>	<b>601.39</b>	<b>601.39</b>	60	3	601.39
E-n51-k5-s2-4-17-46	50	4	4	5						<b>601.39</b>	<b>601.39</b>	<b>601.39</b>	<b>601.39</b>	60	4	601.39
E-n51-k5-s27-47	50	2	3	5						<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	<b>530.76</b>	60	3	530.76
E-n51-k5-s32-37	50	2	3	5						<b>752.59</b>	<b>752.59</b>	<b>752.59</b>	<b>752.59</b>	60	5	752.59
E-n51-k5-s4-46	50	2	3	5						<b>702.33</b>	<b>702.33</b>	<b>702.33</b>	<b>702.33</b>	60	4	702.33
E-n51-k5-s6-12	50	2	3	5						<b>567.42</b>	<b>567.42</b>	<b>567.42</b>	<b>567.42</b>	60	5	567.42
E-n51-k5-s6-12-32-37	50	4	4	5						<b>567.42</b>	<b>567.42</b>	<b>567.42</b>	<b>567.42</b>	60	6	567.42
Avg.										<b>607.94</b>	<b>607.94</b>	<b>607.94</b>	<b>607.94</b>	60	4	607.94

<sup>1</sup> included in Hemmelmayr (2013)

<sup>2</sup> included in Beasley (2014)

Table 4: Results for Set 3 Instances

Instance	C	S	T	CF	HCC			ZXXS		LNS-2E					BKS
					Avg. 5	t(s)	t*(s)	Avg. 5	t*(s)	Avg. 5	Best 5	Best	t(s)	t*(s)	
Set 3a <sup>1,2</sup>															
E-n22-k4-s13-14	21	2	3	4	526.15	43	0	526.15	0	526.15	526.15	526.15	60	2	526.15*
E-n22-k4-s13-16	21	2	3	4	521.09	44	0	521.09	0	521.09	521.09	521.09	60	2	521.09*
E-n22-k4-s13-17	21	2	3	4	496.38	49	0	496.38	0	496.38	496.38	496.38	60	1	496.38*
E-n22-k4-s14-19	21	2	3	4	498.80	43	0	498.80	0	498.80	498.80	498.80	60	1	498.80*
E-n22-k4-s17-19	21	2	3	4	512.81	26	0	512.81	0	512.80	512.80	512.80	60	4	512.80*
E-n22-k4-s19-21	21	2	3	4	520.42	34	0	520.42	0	520.42	520.42	520.42	60	2	520.42*
E-n33-k4-s16-22	32	2	3	4	672.17	76	3	672.17	0	672.17	672.17	672.17	60	4	672.17*
E-n33-k4-s16-24	32	2	3	4	666.02	77	0	666.02	0	666.02	666.02	666.02	60	1	666.02*
E-n33-k4-s19-26	32	2	3	4	680.36	84	0	680.36	0	680.36	680.36	680.36	60	1	680.36*
E-n33-k4-s22-26	32	2	3	4	680.37	77	0	680.37	0	680.36	680.36	680.36	60	1	680.36*
E-n33-k4-s24-28	32	2	3	4	670.43	88	0	670.43	0	670.43	670.43	670.43	60	2	670.43*
E-n33-k4-s25-28	32	2	3	4	650.58	63	0	650.58	0	650.58	650.58	650.58	60	1	650.58*
Avg.					591.30	59	0	591.30	0	591.30	591.30	591.30	60	2	591.30
Set 3b <sup>1</sup>															
E-n51-k5-s12-18	50	2	3	5	690.59	147	4	690.59	1	690.59	690.59	690.59	60	8	690.59
E-n51-k5-s12-41	50	2	3	5	683.05	133	38	683.05	1	683.05	683.05	683.05	60	11	683.05
E-n51-k5-s12-43	50	2	3	5	710.41	217	1	710.41	1	710.41	710.41	710.41	60	5	710.41
E-n51-k5-s39-41	50	2	3	5	728.54	155	18	728.54	4	728.54	728.54	728.54	60	7	728.54
E-n51-k5-s40-41	50	2	3	5	723.75	154	17	723.75	3	723.75	723.75	723.75	60	5	723.75
E-n51-k5-s40-43	50	2	3	5	752.15	158	15	752.15	9	752.15	752.15	752.15	60	12	752.15
Avg.					714.75	161	16	714.75	3	714.75	714.75	714.75	60	8	714.75
Set 3c <sup>2</sup>															
E-n51-k5-s13-19	50	2	3	5						560.73	560.73	560.73	60	10	560.73*
E-n51-k5-s13-42	50	2	3	5						564.45	564.45	564.45	60	3	564.45*
E-n51-k5-s13-44	50	2	3	5						564.45	564.45	564.45	60	2	564.45*
E-n51-k5-s40-42	50	2	3	5						746.31	746.31	746.31	60	5	746.31*
E-n51-k5-s41-42	50	2	3	5						771.56	771.56	771.56	60	15	771.56*
E-n51-k5-s41-44	50	2	3	5						802.91	802.91	802.91	60	16	802.91*
Avg.										668.40	668.40	668.40	60	9	668.40

<sup>1</sup> included in Hemmelmayr (2013)<sup>2</sup> included in Beasley (2014)

Table 5: Results for Set 4a Instances (with constraint on the number of city freighters per satellite)

LNS-2E												
Inst.	C	S	T	CF	Avg. 5	Best 5	Best	t(s)	t*(s)	BKS		
Set 4a												
1	50	2	3	6	1569.42	1569.42	1569.42	60	4	1569.42*		
2	50	2	3	6	1438.32	1438.32	1438.32	60	16	1438.33*		
3	50	2	3	6	1570.43	1570.43	1570.43	60	8	1570.43*		
4	50	2	3	6	1424.04	1424.04	1424.04	60	7	1424.04*		
5	50	2	3	6	2193.52	2193.52	2193.52	60	10	2193.52*		
6	50	2	3	6	1279.89	1279.89	1279.89	60	0	1279.87*		
7	50	2	3	6	1458.60	1458.60	1458.60	60	2	1458.63		
8	50	2	3	6	1363.76	1363.76	1363.76	60	29	1363.74*		
9	50	2	3	6	1450.25	1450.25	1450.25	60	5	1450.27*		
10	50	2	3	6	1407.65	1407.65	1407.65	60	6	1407.64*		
11	50	2	3	6	2052.21	2047.43	2047.43	60	3	2047.46*		
12	50	2	3	6	1209.46	1209.46	1209.46	60	8	1209.42*		
13	50	2	3	6	1481.80	1481.80	1481.80	60	7	1481.83*		
14	50	2	3	6	1393.64	1393.64	1393.64	60	1	1393.61*		
15	50	2	3	6	1489.92	1489.92	1489.92	60	16	1489.94*		
16	50	2	3	6	1389.20	1389.20	1389.20	60	2	1389.17*		
17	50	2	3	6	2088.48	2088.48	2088.48	60	15	2088.49*		
18	50	2	3	6	1227.68	1227.68	1227.68	60	1	1227.61*		
19	50	3	3	6	1564.66	1564.66	1564.66	60	3	1564.66*		
20	50	3	3	6	1272.98	1272.98	1272.98	60	25	1272.97*		
21	50	3	3	6	1577.82	1577.82	1577.82	60	2	1577.82*		
22	50	3	3	6	1281.83	1281.83	1281.83	60	3	1281.83*		
23	50	3	3	6	1807.35	1807.35	1807.35	60	8	1807.35*		
24	50	3	3	6	1282.69	1282.69	1282.69	60	0	1282.68*		
25	50	3	3	6	1522.40	1522.40	1522.40	60	4	1522.42*		
26	50	3	3	6	1167.47	1167.47	1167.47	60	1	1167.46*		
27	50	3	3	6	1481.56	1481.56	1481.56	60	42	1481.57*		
28	50	3	3	6	1210.46	1210.46	1210.46	60	3	1210.44*		
29	50	3	3	6	1722.06	1722.00	1722.00	60	31	1722.04		
30	50	3	3	6	1211.63	1211.63	1211.63	60	12	1211.59*		
31	50	3	3	6	1490.32	1490.32	1490.32	60	7	1490.34		
32	50	3	3	6	1199.05	1199.05	1199.05	60	24	1199.00*		
33	50	3	3	6	1508.32	1508.32	1508.32	60	14	1508.30		
34	50	3	3	6	1233.96	1233.96	1233.96	60	7	1233.92*		
35	50	3	3	6	1718.42	1718.42	1718.42	60	41	1718.41		
36	50	3	3	6	1228.95	1228.95	1228.95	60	0	1228.89*		
37	50	5	3	6	1528.73	1528.73	1528.73	60	25	1528.73*		
38	50	5	3	6	1169.20	1169.20	1169.20	60	15	1169.20*		
39	50	5	3	6	1520.92	1520.92	1520.92	60	17	1520.92*		
40	50	5	3	6	1199.42	1199.42	1199.42	60	2	1199.42*		
41	50	5	3	6	1667.96	1667.96	1667.96	60	9	1667.96*		
42	50	5	3	6	1194.54	1194.54	1194.54	60	19	1194.54*		
43	50	5	3	6	1439.67	1439.67	1439.67	60	14	1439.67*		
44	50	5	3	6	1045.14	1045.14	1045.14	60	24	1045.13*		
45	50	5	3	6	1451.48	1450.95	1450.95	60	3	1450.96*		
46	50	5	3	6	1088.79	1088.79	1088.79	60	2	1088.77*		
47	50	5	3	6	1587.29	1587.29	1587.29	60	15	1587.29*		
48	50	5	3	6	1082.21	1082.21	1082.21	60	23	1082.20*		
49	50	5	3	6	1434.88	1434.88	1434.88	60	14	1434.88*		
50	50	5	3	6	1083.16	1083.16	1083.16	60	23	1083.12*		
51	50	5	3	6	1398.03	1398.03	1398.03	60	21	1398.05*		
52	50	5	3	6	1125.69	1125.69	1125.69	60	6	1125.67*		
53	50	5	3	6	1567.79	1567.79	1567.79	60	21	1567.77*		
54	50	5	3	6	1127.66	1127.66	1127.66	60	15	1127.61*		
Avg.					1420.05	1419.95	1419.95	60	12	1419.94		

Table 6: Results for Set 4b Instances ( $v_s^2 = v^2$ )

Inst.	C	S	T	CF	HCC				ZXXS				LNS-2E						BKS
					Avg. 5	Best	t(s)	t*(s)	Avg. 5	Best	t*(s)	Avg. 5	Best 5	Best	t(s)	t*(s)			
Set 4b																			
1	50	2	3	6	1569.42	1569.42	235	6	1569.42	1569.42	1	1569.42	1569.42	1569.42	60	15	1569.42*		
2	50	2	3	6	1441.02	1438.33	155	43	1438.33	1438.33	40	1438.32	1438.32	1438.32	60	6	1438.33*		
3	50	2	3	6	1570.43	1570.43	183	3	1570.43	1570.43	1	1570.43	1570.43	1570.43	60	7	1570.43*		
4	50	2	3	6	1424.04	1424.04	130	11	1429.04	1424.04	73	1424.04	1424.04	1424.04	60	7	1424.04*		
5	50	2	3	6	2194.11	2194.11	614	63	2193.52	2193.52	34	2193.52	2193.52	2193.52	60	18	2193.52*		
6	50	2	3	6	1279.87	1279.87	99	2	1279.87	1279.87	1	1279.89	1279.89	1279.89	60	0	1279.87*		
7	50	2	3	6	1458.63	1458.63	169	6	1408.57	1408.57	16	1408.58	1408.58	1408.58	60	17	1408.57*		
8	50	2	3	6	1360.32	1360.32	205	5	1360.32	1360.32	4	1360.32	1360.32	1360.32	60	8	1360.32*		
9	50	2	3	6	1450.27	1450.27	204	46	1403.53	1403.53	4	1403.53	1403.53	1403.53	60	11	1403.53*		
10	50	2	3	6	1360.56	1360.56	174	1	1360.56	1360.56	1	1360.54	1360.54	1360.54	60	1	1360.56*		
11	50	2	3	6	2059.88	2059.88	648	101	2059.41	2059.41	4	2054.60	2047.43	2047.43	60	2	2047.46*		
12	50	2	3	6	1209.42	1209.42	205	44	1209.42	1209.42	6	1209.46	1209.46	1209.46	60	20	1209.42*		
13	50	2	3	6	1481.83	1481.83	220	25	1450.93	1450.93	2	1450.95	1450.94	1450.94	60	10	1450.93*		
14	50	2	3	6	1393.61	1393.61	189	6	1393.61	1393.61	1	1393.64	1393.64	1393.64	60	3	1393.61*		
15	50	2	3	6	1489.94	1489.94	173	9	1466.83	1466.83	1	1466.84	1466.84	1466.84	60	2	1466.83*		
16	50	2	3	6	1387.83	1387.83	147	6	1387.83	1387.83	6	1387.85	1387.85	1387.85	60	12	1387.83*		
17	50	2	3	6	2088.49	2088.49	625	165	2088.49	2088.49	27	2088.48	2088.48	2088.48	60	13	2088.49*		
18	50	2	3	6	1227.61	1227.61	94	3	1227.61	1227.61	1	1227.68	1227.68	1227.68	60	5	1227.61*		
19	50	3	3	6	1546.28	1546.28	171	25	1546.28	1546.28	25	1546.28	1546.28	1546.28	60	34	1546.28*		
20	50	3	3	6	1272.97	1272.97	99	12	1272.97	1272.97	57	1272.98	1272.98	1272.98	60	11	1272.97*		
21	50	3	3	6	1577.82	1577.82	155	61	1577.82	1577.82	19	1577.82	1577.82	1577.82	60	16	1577.82*		
22	50	3	3	6	1281.83	1281.83	127	2	1281.83	1281.83	28	1281.83	1281.83	1281.83	60	4	1281.83*		
23	50	3	3	6	1652.98	1652.98	175	5	1652.98	1652.98	3	1652.98	1652.98	1652.98	60	4	1652.98*		
24	50	3	3	6	1282.68	1282.68	110	2	1282.68	1282.68	1	1282.69	1282.69	1282.69	60	1	1282.68*		
25	50	3	3	6	1440.84	1440.68	154	53	1408.57	1408.57	17	1408.58	1408.58	1408.58	60	20	1408.57*		
26	50	3	3	6	1167.46	1167.46	96	0	1167.46	1167.46	7	1167.47	1167.47	1167.47	60	16	1167.46*		
27	50	3	3	6	1447.79	1444.50	163	12	1454.63	1444.51	39	1444.49	1444.49	1444.49	60	20	1444.50*		
28	50	3	3	6	1210.44	1210.44	143	7	1210.44	1210.44	2	1210.46	1210.46	1210.46	60	6	1210.44*		
29	50	3	3	6	1561.81	1559.82	178	102	1555.56	1552.66	30	1552.66	1552.66	1552.66	60	39	1552.66*		
30	50	3	3	6	1211.59	1211.59	132	5	1211.59	1211.59	1	1211.63	1211.63	1211.63	60	5	1211.59*		
31	50	3	3	6	1440.86	1440.86	144	37	1441.07	1440.86	29	1440.85	1440.85	1440.85	60	15	1440.86*		
32	50	3	3	6	1199.00	1199.00	102	11	1199.00	1199.00	10	1199.05	1199.05	1199.05	60	21	1199.00*		
33	50	3	3	6	1478.86	1478.86	159	16	1478.86	1478.86	14	1478.87	1478.87	1478.87	60	15	1478.86*		
34	50	3	3	6	1233.92	1233.92	93	4	1233.92	1233.92	11	1233.96	1233.96	1233.96	60	11	1233.92*		
35	50	3	3	6	1570.80	1570.72	182	116	1570.72	1570.72	4	1570.73	1570.73	1570.73	60	10	1570.72*		
36	50	3	3	6	1228.89	1228.89	123	6	1228.89	1228.89	2	1228.95	1228.95	1228.95	60	6	1228.89*		
37	50	5	3	6	1528.81	1528.73	143	55	1528.98	1528.73	43	1528.73	1528.73	1528.73	60	27	1528.73		
38	50	5	3	6	1163.07	1163.07	88	15	1163.07	1163.07	1	1163.07	1163.07	1163.07	60	17	1163.07*		
39	50	5	3	6	1520.92	1520.92	158	33	1520.92	1520.92	16	1520.92	1520.92	1520.92	60	25	1520.92*		
40	50	5	3	6	1165.24	1163.04	84	20	1163.04	1163.04	7	1163.04	1163.04	1163.04	60	5	1163.04*		
41	50	5	3	6	1652.98	1652.98	150	12	1652.98	1652.98	9	1652.98	1652.98	1652.98	60	14	1652.98*		
42	50	5	3	6	1190.17	1190.17	95	31	1190.17	1190.17	71	1190.17	1190.17	1190.17	60	9	1190.17*		
43	50	5	3	6	1408.95	1406.11	151	60	1406.11	1406.11	24	1407.09	1406.10	1406.10	60	13	1406.11*		
44	50	5	3	6	1035.32	1035.03	109	30	1035.03	1035.03	7	1035.05	1035.05	1035.05	60	42	1035.03*		
45	50	5	3	6	1406.43	1403.10	144	104	1402.03	1402.03	27	1401.87	1401.87	1401.87	60	21	1401.87*		
46	50	5	3	6	1058.97	1058.11	74	17	1058.11	1058.11	7	1058.10	1058.10	1058.10	60	16	1058.11*		
47	50	5	3	6	1564.41	1559.82	185	103	1557.04	1552.66	25	1552.66	1552.66	1552.66	60	24	1552.66*		
48	50	5	3	6	1074.50	1074.50	83	2	1074.50	1074.50	1	1074.51	1074.51	1074.51	60	4	1074.50*		
49	50	5	3	6	1435.28	1434.88	140	81	1434.88	1434.88	38	1434.88	1434.88	1434.88	60	9	1434.88		
50	50	5	3	6	1065.25	1065.25	92	16	1065.25	1065.25	2	1065.30	1065.30	1065.30	60	33	1065.25*		
51	50	5	3	6	1387.72	1387.51	138	46	1387.51	1387.51	3	1387.51	1387.51	1387.51	60	19	1387.51*		
52	50	5	3	6	1103.76	1103.42	102	47	1103.42	1103.42	22	1103.47	1103.47	1103.47	60	17	1103.42*		
53	50	5	3	6	1545.73	1545.73	148	37	1545.73	1545.73	4	1545.76	1545.76	1545.76	60	21	1545.73*		
54	50	5	3	6	1113.62	1113.62	90	2	1113.62	1113.62	12	1113.66	1113.66	1113.66	60	5	1113.62*		
Avg.					1401.39	1400.96	169	32	1397.69	1397.27	16	1397.21	1397.06	1397.06	60	14	1397.04		



Table 7: Results for Set 5 Instances

Instance	C	S	T	CF	HCC				LNS-2E					BKS	
					Avg. 5	Best	t(s)	t*(s)	Avg. 5	Best 5	Best	t(s)	t*(s)		
Set 5															
100-5-1	100	5	5	32	1588.73	1565.45	353	116	1566.87	<b>1564.46</b>	<b>1564.46</b>	900	201	1564.46*	
100-5-1b	100	5	5	15	1126.93	1111.34	397	45	1111.93	<b>1108.62</b>	<b>1108.62</b>	900	176	<u>1108.62</u>	
100-5-2	100	5	5	32	1022.29	<b>1016.32</b>	406	117	1017.94	<b>1016.32</b>	<b>1016.32</b>	900	75	1016.32*	
100-5-2b	100	5	5	15	789.05	<b>782.25</b>	340	170	783.07	<b>782.25</b>	<b>782.25</b>	900	152	782.25	
100-5-3	100	5	5	30	1046.67	<b>1045.29</b>	352	80	<b>1045.29</b>	<b>1045.29</b>	<b>1045.29</b>	900	131	1045.29*	
100-5-3b	100	5	5	16	828.99	828.99	391	127	<b>828.54</b>	<b>828.54</b>	<b>828.54</b>	900	62	<u>828.54</u>	
100-10-1	100	10	5	35	1137.00	1130.23	429	136	1132.11	1125.53	<b>1124.93</b>	900	567	<u>1124.93</u>	
100-10-1b	100	10	5	18	928.01	916.48	476	262	922.85	<b>916.25</b>	<b>916.25</b>	900	424	<u>916.25</u>	
100-10-2	100	10	5	33	1009.49	<b>990.58</b>	356	232	1014.61	1012.14	1002.15	900	471	990.58	
100-10-2b	100	10	5	18	773.58	<b>768.61</b>	432	157	786.64	781.27	774.11	900	416	768.61	
100-10-3	100	10	5	32	1055.28	<b>1043.25</b>	415	209	1053.55	1049.77	1048.53	900	105	1043.25	
100-10-3b	100	10	5	17	861.88	<b>850.92</b>	418	29	858.72	854.90	854.90	900	175	850.92	
200-10-1	200	10	5	62	1626.83	1574.12	888	207	1598.46	1580.34	<b>1556.79</b>	900	730	<u>1556.79</u>	
200-10-1b	200	10	5	30	1239.79	1201.75	692	374	1217.23	1191.59	<b>1187.62</b>	900	588	<u>1187.62</u>	
200-10-2	200	10	5	63	1416.87	1374.74	1072	496	1406.16	1366.36	<b>1365.74</b>	900	534	<u>1365.74</u>	
200-10-2b	200	10	5	30	1018.57	1003.75	1058	221	1016.05	1008.46	<b>1002.85</b>	900	721	<u>1002.85</u>	
200-10-3	200	10	5	63	1808.24	<b>1787.73</b>	916	305	1809.44	1797.80	1793.99	900	675	1787.73	
200-10-3b	200	10	5	30	1208.38	1200.74	1217	478	1206.85	1202.21	<b>1197.90</b>	900	523	<u>1197.90</u>	
Avg.					1138.14	1121.81	589	209	1132.02	1124.01	1120.62	900	374	1118.81	

Table 8: Results for Set 6a Instances (no handling costs)

Instance	C	S	T	CF	LNS-2E						BKS
					Avg. 5	Best 5	Best	t(s)	t*(s)		
Set 6a											
A-n51-4	50	4	2	50	<b>652.00</b>	<b>652.00</b>	<b>652.00</b>	60	19	652.00*	
A-n51-5	50	5	2	50	<b>663.41</b>	<b>663.41</b>	<b>663.41</b>	60	37	663.41*	
A-n51-6	50	6	2	50	<b>662.51</b>	<b>662.51</b>	<b>662.51</b>	60	23	662.51*	
A-n76-4	75	4	3	75	985.98	<b>985.95</b>	<b>985.95</b>	900	128	985.95*	
A-n76-5	75	5	3	75	981.19	<b>979.15</b>	<b>979.15</b>	900	286	979.15*	
A-n76-6	75	6	3	75	971.65	<b>970.20</b>	<b>970.20</b>	900	233	970.20*	
A-n101-4	100	4	4	100	1194.38	<b>1194.17</b>	<b>1194.17</b>	900	267	1194.17*	
A-n101-5	100	5	4	100	1215.89	1211.40	1211.40	900	414	1211.38*	
A-n101-6	100	6	4	100	1161.91	1158.97	<b>1155.96</b>	900	154	<u>1155.96</u>	
B-n51-4	50	4	2	50	<b>563.98</b>	<b>563.98</b>	<b>563.98</b>	60	6	563.98*	
B-n51-5	50	5	2	50	<b>549.23</b>	<b>549.23</b>	<b>549.23</b>	60	55	549.23*	
B-n51-6	50	6	2	50	<b>556.32</b>	<b>556.32</b>	<b>556.32</b>	60	39	556.32*	
B-n76-4	75	4	3	75	793.97	<b>792.73</b>	<b>792.73</b>	900	320	792.73*	
B-n76-5	75	5	3	75	784.27	784.19	784.19	900	190	783.93*	
B-n76-6	75	6	3	75	775.75	774.24	<b>774.17</b>	900	160	774.17*	
B-n101-4	100	4	4	100	939.79	<b>939.21</b>	<b>939.21</b>	900	377	939.21*	
B-n101-5	100	5	4	100	971.27	969.13	969.13	900	161	967.82*	
B-n101-6	100	6	4	100	961.91	<b>960.29</b>	<b>960.29</b>	900	88	960.29*	
C-n51-4	50	4	2	50	<b>689.18</b>	<b>689.18</b>	<b>689.18</b>	60	13	689.18*	
C-n51-5	50	5	2	50	<b>723.12</b>	<b>723.12</b>	<b>723.12</b>	60	19	723.12*	
C-n51-6	50	6	2	50	<b>697.00</b>	<b>697.00</b>	<b>697.00</b>	60	46	697.00*	
C-n76-4	75	4	3	75	1055.61	<b>1054.89</b>	<b>1054.89</b>	900	339	1054.89*	
C-n76-5	75	5	3	75	<b>1115.32</b>	<b>1115.32</b>	<b>1115.32</b>	900	113	1115.32*	
C-n76-6	75	6	3	75	1066.88	<b>1060.52</b>	<b>1060.52</b>	900	474	<u>1060.52</u>	
C-n101-4	100	4	4	100	1305.94	<b>1302.16</b>	<b>1302.16</b>	900	236	<u>1302.16</u>	
C-n101-5	100	5	4	100	1307.24	1306.27	<b>1305.82</b>	900	141	<u>1305.82</u>	
C-n101-6	100	6	4	100	1292.10	<b>1284.48</b>	<b>1284.48</b>	900	446	1284.48	
Avg.					912.51	911.11	910.98	620	177	910.92	

Table 9: Results for Set 6b Instances ( $h_s \neq 0$ )

Instance	C	S	T	CF	LNS-2E						BKS
					Avg. 5	Best 5	Best	t(s)	t*(s)		
Set 6b											
A-n51-4	50	4	2	50	744.24	744.24	744.24	60	17	744.24*	
A-n51-5	50	5	2	50	811.51	811.51	811.51	60	49	811.52*	
A-n51-6	50	6	2	50	930.11	930.11	930.11	60	31	930.11*	
A-n76-4	75	4	3	75	1385.51	1385.51	1385.51	900	26	1385.51*	
A-n76-5	75	5	3	75	1519.86	1519.86	1519.86	900	71	1519.86*	
A-n76-6	75	6	3	75	1666.28	1666.06	1666.06	900	533	1666.06*	
A-n101-4	100	4	4	100	1884.48	1883.79	1883.79	900	283	1881.44*	
A-n101-5	100	5	4	100	1723.06	1714.58	1711.95	900	318	1709.06	
A-n101-6	100	6	4	100	1795.36	1793.76	1791.44	900	100	1777.69	
B-n51-4	50	4	2	50	653.09	653.09	653.09	60	21	653.09*	
B-n51-5	50	5	2	50	672.10	672.10	672.10	60	51	672.10*	
B-n51-6	50	6	2	50	767.13	767.13	767.13	60	58	767.13*	
B-n76-4	75	4	3	75	1094.52	1094.52	1094.52	900	44	1094.52*	
B-n76-5	75	5	3	75	1218.12	1218.12	1218.12	900	19	1218.13*	
B-n76-6	75	6	3	75	1328.90	1328.90	1326.76	900	329	1326.76*	
B-n101-4	100	4	4	100	1500.80	1500.55	1500.55	900	82	1500.55	
B-n101-5	100	5	4	100	1398.05	1398.05	1395.32	900	317	1395.32	
B-n101-6	100	6	4	100	1455.05	1453.54	1453.54	900	460	1450.39	
C-n51-4	50	4	2	50	866.58	866.58	866.58	60	22	866.58*	
C-n51-5	50	5	2	50	943.12	943.12	943.12	60	12	943.12*	
C-n51-6	50	6	2	50	1050.42	1050.42	1050.42	60	18	1050.42*	
C-n76-4	75	4	3	75	1439.39	1438.96	1438.96	900	84	1438.96*	
C-n76-5	75	5	3	75	1745.49	1745.39	1745.39	900	281	1745.39*	
C-n76-6	75	6	3	75	1759.40	1756.54	1756.54	900	358	1756.54*	
C-n101-4	100	4	4	100	2076.36	2073.84	2064.86	900	378	2064.86	
C-n101-5	100	5	4	100	1974.39	1967.51	1964.63	900	285	1964.63	
C-n101-6	100	6	4	100	1867.45	1861.50	1861.50	900	401	1861.50	
Avg.					1343.36	1342.20	1341.39	620	172	1340.57	

To test the robustness of our algorithm we applied it also to the 2ELRPSD instances without any further changing of operators or tuning. The results can be seen in Tables 10 and 11. BKS are derived from Schwengerer et al. (2012); Nguyen et al. (2012a,b); Contardo et al. (2012). Also on this problem class LNS-2E is competitive, with solutions being on average within 0.6% of the solutions found by the state-of-the-art VNS by Schwengerer et al. (2012) (SPR).

Table 10: Results for 2ELRPSD Instances Set Nguyen

Instance	C	S	SPR				LNS-2E				BKS	
			Avg. 20	Best 20	t(s)	t*(s)	Avg. 20	Best 20	Best	t(s)		t*(s)
Set Nguyen												
25-5N	25	5	80370.00	80370	76	3	80370.00	80370	80370	60	5	80370
25-5Nb	25	5	64562.00	64562	91	0	64562.00	64562	64562	60	16	64562
25-5MN	25	5	78947.00	78947	61	1	78947.00	78947	78947	60	6	78947
25-5MNb	25	5	64438.00	64438	89	0	64438.00	64438	64438	60	4	64438
50-5N	50	5	137815.00	137815	116	35	137815.00	137815	137815	60	25	137815
50-5Nb	50	5	110204.40	110094	132	51	110981.85	110094	110094	60	22	110094
50-5MN	50	5	123484.00	123484	125	41	123484.00	123484	123484	60	4	123484
50-5MNb	50	5	105687.00	105401	202	48	105783.45	105401	105401	60	16	105401
50-10N	50	10	115725.00	115725	143	24	117325.55	115725	115725	60	20	115725
50-10Nb	50	10	87345.00	87315	176	66	88212.00	87520	87315	60	23	87315
50-10MN	50	10	135519.00	135519	144	10	138241.35	135519	135519	60	14	135519
50-10MNb	50	10	110613.00	110613	218	13	111520.80	110613	110613	60	22	110613
100-5N	100	5	200685.05	193228	168	101	193806.85	193229	193229	900	239	193228
100-5Nb	100	5	164508.10	158927	258	144	159064.10	158927	158927	900	315	158927
100-5MN	100	5	206567.40	204682	184	159	204876.10	204682	204682	900	105	204682
100-5MNb	100	5	166357.35	165744	315	247	165795.35	165744	165744	900	252	165744
100-10N	100	10	214585.60	209952	223	167	216265.50	210799	209952	900	344	209952
100-10Nb	100	10	155790.60	155489	352	251	161273.30	155489	155489	900	442	155489
100-10MN	100	10	203798.05	201275	229	163	204396.15	201275	201275	900	324	201275
100-10MNb	100	10	170791.25	170625	347	283	172202.45	170625	170625	900	268	170625
200-10N	200	10	349584.15	345267	641	525	359948.65	350680	350680	900	758	345267
200-10Nb	200	10	264228.90	256171	907	791	260698.20	257191	257191	900	748	256171
200-10MN	200	10	332207.50	323801	453	441	329486.45	324279	324279	900	777	323801
200-10MNb	200	10	292036.65	287076	944	843	297857.50	293339	290702	900	778	287076
Avg.			163993.75	161938	275	184	164472.98	162531	162377	480	230	161938

Table 11: Results for 2ELRPD Instances Set Prodhon

Instance	C	S	SPR				LNS-2E					BKS
			Avg. 20	Best 20	t(s)	t*(s)	Avg. 20	Best 20	Best	t(s)	t*(s)	
Set Prodhon												
20-5-1	20	5	89075.00	89075	63	2	89075.00	89075	89075	60	4	89075
20-5-1b	20	5	61863.00	61863	83	0	61863.00	61863	61863	60	2	61863
20-5-2	20	5	84489.50	84478	62	11	84478.00	84478	84478	60	19	84478
20-5-2b	20	5	61033.80	60838	125	0	60838.00	60838	60838	60	5	60838
50-5-1	50	5	130859.30	130843	80	16	131454.00	131085	130843	60	24	130843
50-5-1b	50	5	101548.00	101530	128	35	101669.20	101530	101530	60	14	101530
50-5-2	50	5	131825.00	131825	97	11	131827.00	131825	131825	60	16	131825
50-5-2b	50	5	110332.00	110332	198	12	110332.00	110332	110332	60	14	110332
50-5-2BIS	50	5	122599.00	122599	112	91	122599.00	122599	122599	60	3	122599
50-5-2bBIS	50	5	105935.50	105696	198	155	105707.85	105696	105696	60	18	105696
50-5-3	50	5	128436.00	128379	80	9	128614.50	128379	128379	60	10	128379
50-5-3b	50	5	104006.00	104006	131	6	104006.00	104006	104006	60	7	104006
100-5-1	100	5	318667.00	318225	226	153	319268.60	318399	318399	900	303	318134
100-5-1b	100	5	257436.35	256991	301	220	257686.40	256991	256888	900	268	256878
100-5-2	100	5	231340.00	231305	204	131	231488.85	231305	231305	900	229	231305
100-5-2b	100	5	194812.70	194763	240	202	194800.35	194763	194729	900	176	194728
100-5-3	100	5	245334.90	244470	174	124	245178.75	244071	244071	900	377	244071
100-5-3b	100	5	195586.20	195381	180	111	195123.20	194110	194110	900	327	194110
100-10-1	100	10	357381.40	352694	233	167	362648.70	354525	352122	900	401	351243
100-10-1b	100	10	300239.15	298186	299	194	312451.60	299758	298298	900	540	297167
100-10-2	100	10	304931.20	304507	248	194	307937.60	304909	304438	900	520	304438
100-10-2b	100	10	264592.00	264092	307	208	265814.85	264173	263876	900	310	263873
100-10-3	100	10	312701.25	311447	227	141	318952.10	311699	310930	900	512	310200
100-10-3b	100	10	261577.90	260516	303	218	265442.40	262932	261566	900	437	260328
200-10-1	200	10	552488.90	548730	1009	748	564159.80	550672	550672	900	725	548703
200-10-1b	200	10	448095.45	445791	635	576	456952.40	448188	447113	900	692	445301
200-10-2	200	10	513673.40	497451	1158	832	499499.45	498486	498397	900	656	497451
200-10-2b	200	10	432687.00	422668	730	696	428912.35	422967	422877	900	601	422668
200-10-3	200	10	529578.00	527162	970	903	568539.15	534271	533174	900	668	527162
200-10-3b	200	10	404431.25	402117	592	557	425078.20	417686	417429	900	700	401672
Avg.			245251.87	243599	313	224	248413.28	244720	244395	564	286	243363

Finding high-quality solutions is increasingly difficult as the problem size grows, and different instance characteristics influence solution quality. Figure 2 displays boxplots of 2EVRP instances grouped together by number of customers (2a), or number of satellites (2b), similar customer distribution (2c) and similar satellite distribution (2d) using gaps of the average value of five runs to BKS.

Problem difficulty quickly grows with the number of satellites, as well as with the number of customers, as plotted in the upper part of Figure 2. The number of samples for the different classes varies a lot: There are only 18 instances with 75 customers, but 165 instances with 50 customers, which explains the large number of outliers for those instances. Crainic et al. (2010) provide a detailed overview on the distribution of customers and satellites in instances of Set 4. There are three distribution patterns for customers: *random*, with equally distributed nodes; *centroids*, where more customers are located in six centroids in a central zone, and some customers closer together in four outer areas, representing suburbs. In the *quadrant* pattern customers are arranged in conglomerations of higher density in each of the four quadrants. The three patterns for satellites are: *random*, where satellites are randomly placed on a ring around the customers; *sliced*, with the satellites distributed more evenly on the ring around customers; and *forbidden zone*: a random angle on the ring was chosen where no satellites could be used, to simulate various conditions like cities located near lakes or mountains. Figure 2c shows that instances with customers located in centroids are harder to solve. On the other hand, the distribution pattern of the satellites doesn't have a large impact on solution quality.

## 6.5 Graphical Example

Several structurally different 2EVRP solutions can have similar objective values. We discuss and visualise this with the help of a demonstrative graphical example. Also the solution differences for an instance with or without constraining the number of city freighters per satellite are pointed out.

The selection of the correct subset of satellites to use is crucial. A graphical representation of different solutions of the Set 4 instance 38 is provided in Figure 3. The locations of the depot (square), satellites (triangles) and customers (circles) is the same for each solution. Nevertheless, the obtained vehicle routes are substantially different given different subsets of open satellites.

Figure 3a represents the optimal solution to the Set 4a instance 38. A maximum of  $v_s^2 = 2$  city freighters per satellite are available. The total demand of all customers sums up to 20206 units of freight. The capacity  $Q^2$  of a city freighter is 5000 units. Any feasible solution needs at least  $\lceil \frac{20206}{5000} \rceil = 5$  city freighters, and thus at least  $\lceil 5/2 \rceil = 3$  satellites have to be used. Two city freighter routes are located on the left of the figure, and three city freighter routes on the right hand side, where two city freighters leave from the same satellite, and a third one from a close by satellite.

Considering the instance as in Set 4b, with a global number of city freighters available but no constraints on the distribution amongst satellites ( $v_s^2 = v^2$ ), the optimal solution is displayed in Figure 3b. Three city freighters can leave the same satellite, as is the case on the left hand side. Only two of the five satellites have to be used.

LNS-2E starts with the construction of routes at the second level. In early stages of the optimisation process, all customers are likely assigned to their closest satellites. Without neighbourhoods that impact the selection of satellites, the algorithm would likely be trapped in a local optimum such as the one of Figure 3c. If partial routes originating at the bottom right satellite exist, customers may be sequentially inserted in those routes and the bottom left satellite may not be opened. The solution displayed in Figure 3d is often obtained. It has the best cost considering only the second level, but long truck routes on the first level set this gain off, leading to a worse solution overall.

Figure 3e shows the best found solution if only the top right satellite is open. The cost differences from one solution to the next one are small, although the solution itself is fundamentally different. If the two satellites at the bottom are both selected for closure at the same time, then the algorithm finds the optimal solution within seconds.

We tried different strategies to evaluate the chances of a satellite to be included in the best solution: taking into account a delta evaluation on the truck route, or combining this value with the total units shipped through this satellite; or the absolute distance from the depot. We observed that closing satellites randomly is a straightforward and very simple approach, which performs quite well on average over all the different benchmark instances, whereas other techniques present advantages and disadvantages in several special cases.

For further research, we suggest to shift the cost structure towards more realistic scenarios. In the classic 2EVRP as we considered it, the cost of large trucks and small city freighters is the same. For instance Sets 2 to 4, the capacity of a truck is 2.5 times higher than of a city freighter. For Set 5 instances this ratio gets up to more than 14, so one can safely assume that the operating cost of a truck will be higher than of a city freighter in more realistic set-ups. This has not yet been taken into account in previous publications on the 2EVRP, and would lead to large differences between the solution costs of Figure 3.

## 6.6 Sensitivity Analysis

Sensitivity analyses were conducted to evaluate the impact of major parameters and components of the method. In particular, we evaluate the impact of disabling single destroy operators or local search procedures at a time and provide the average objective value over five runs of all benchmark instances. Table 12 shows the average objective values for instances of each class and its average deviation (Gap (%)) when disabling elements of the algorithm.

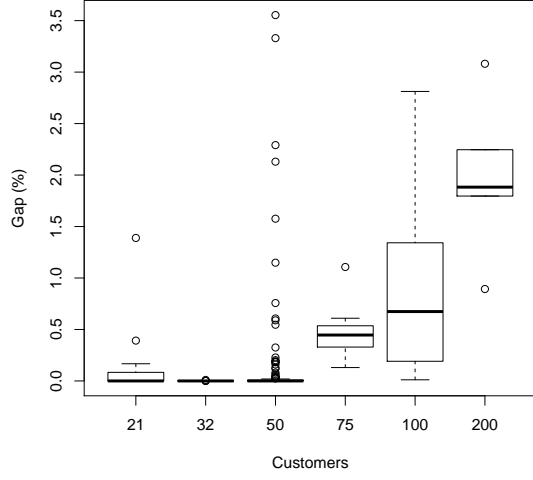
The elimination of the open all satellites (*no open*) neighbourhood has a strong impact on solution quality. As closed satellites are only opened again in a re-start phase, the algorithm is likely to be trapped in a local optimum.

If no satellites are forced to be closed, there is no need to open up any satellite again. In this case, solution quality deteriorates by 1.15% on average (*no close*). For some instances, satellites located very far away from the depot will not be used in the best solution, but on the second level it seems to be beneficial to use them for customers close by. The impact on the 2ELRPSD, especially the instance Set Prodhon is quite strong, as the fixed costs of a satellite are not taken into account on the second level. If a satellite with high opening costs is located close to many customers, it will very likely be used, although it would pay off to use a cheaper one further away. A similar behaviour was discussed in Section 6.5 and Figure 3.

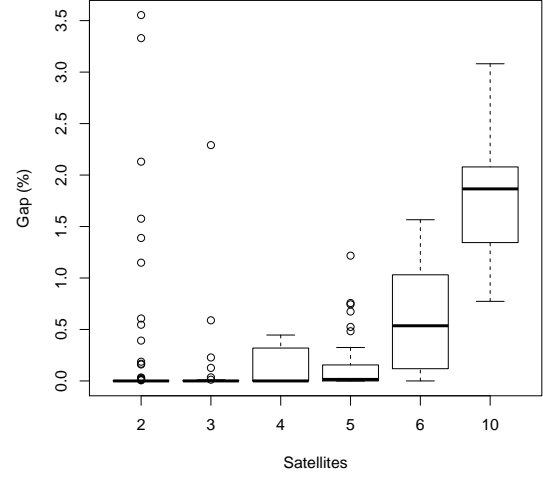
Some techniques work better on smaller instances, while others perform better on the larger instances. For some cases we even observed small improvements when an operator was not used. Not using biased node

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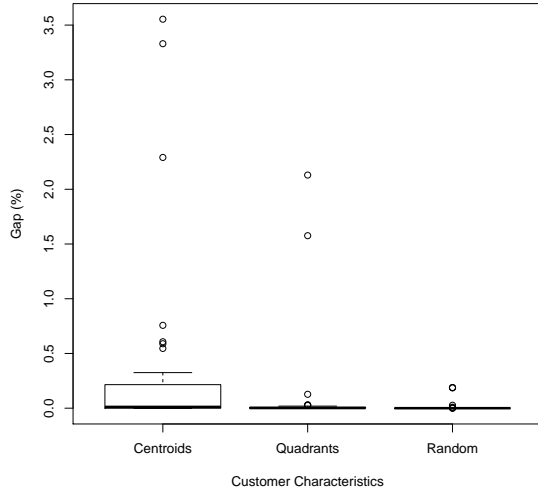
<sup>2</sup>The hinges depict approximately the first and third quartile of the solutions. The whiskers extend to  $\pm 1.58 \frac{Q_3 - Q_1}{\sqrt{n}}$ . (The R Foundation for Statistical Computing, 2014). See the Appendix 8.2 for a detailed definition.



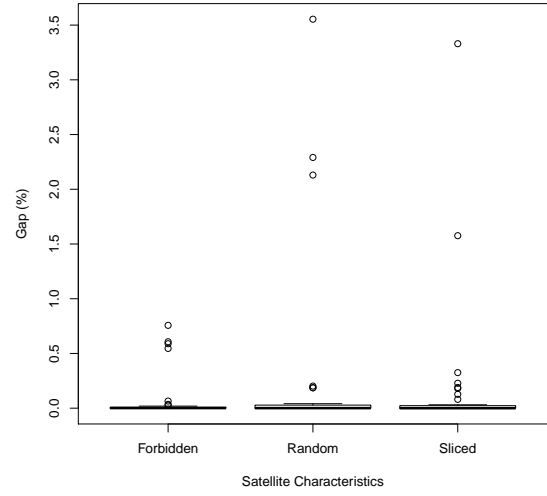
(a) number of Customers



(b) number of Satellites

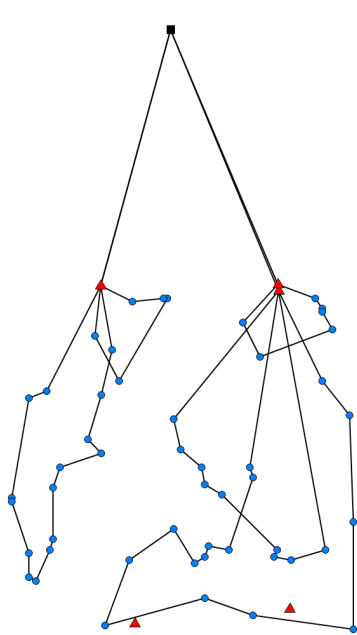


(c) Set 4: Customer distribution

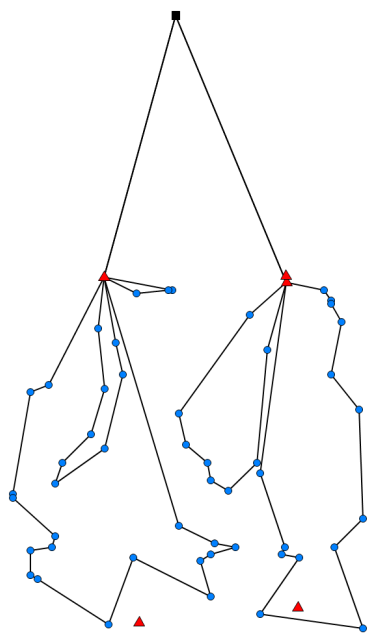


(d) Set 4: Satellite distribution

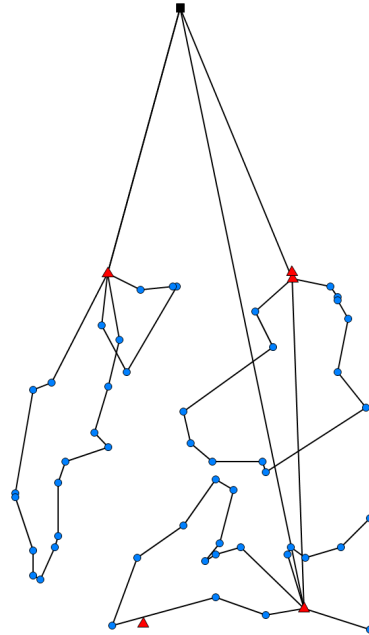
Figure 2: Boxplots<sup>2</sup> of solution quality for instances grouped by number of customers/satellites and distribution characteristics



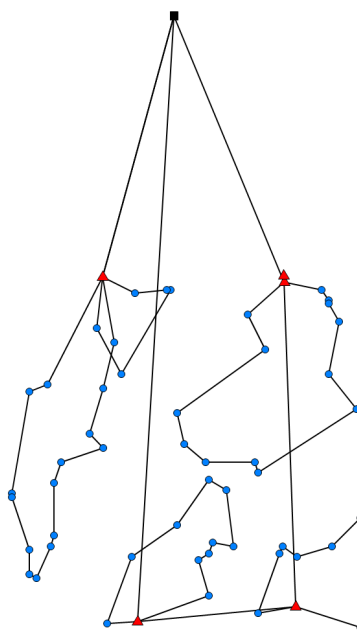
(a) max. 2 CF/Sat, 1169.20\*, optimal for Set 4a



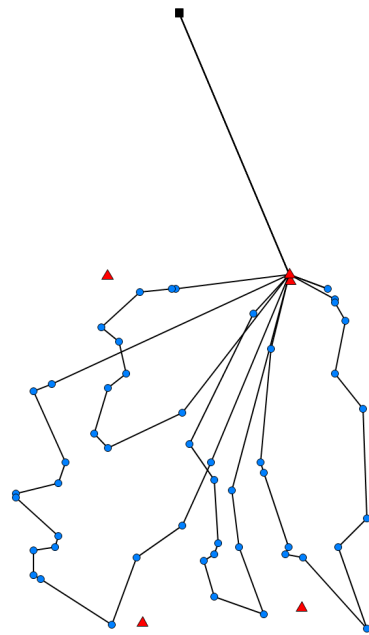
(b) 2 Satellites used, 1163.07\*, optimal for Set 4b



(c) 3 Satellites used, 1210.81



(d) 4 Satellites used, 1226.43



(e) 1 Satellite used, 1255.98

Figure 3: Different Solutions of Instance 38 from Set 4, depending on satellite openings and  $v_s^2$

removal for example is needed to robustly find optimal solutions on the smaller instance sets (2-4) but yields improvements on the larger sets.

For local search techniques, we also observe differences on methods for small or large instance sets respectively. Removing classic 2-opt has a small impact. The repair mechanism finds already high quality single routes in terms of 2-optimality. Eliminating the inter-tour operator 2-opt\* has a stronger effect on larger Set 5 and Set 6 instances, which contain more routes than the smaller instances. The relocate neighbourhood is essential for instances with more than 50 customers, whereas exchanging two nodes against one can have a negative effect on those larger instances. We still decided to keep it in the design of the algorithm, as this local search is needed to find optimal solutions for smaller instances. Of course the algorithm could be fine-tuned for specific applications or instance sizes.

Table 12: Sensitivity Analysis and contribution of individual components

Set	Sensitivity Analysis						
	base	no related	no biased	no route	no single	no close	no open
2	578.27	0.21%	0.02%	0.17%	0.02%	0.07%	2.07%
3	641.44	0.31%	0.06%	0.27%	0.06%	0.07%	1.66%
4a	1420.05	0.25%	0.02%	0.10%	0.04%	0.53%	1.37%
4b	1397.21	0.16%	-0.01%	0.18%	0.05%	1.14%	1.39%
5	1132.02	0.69%	-0.07%	0.31%	0.35%	0.39%	2.35%
6a	912.51	0.25%	-0.03%	0.07%	-0.01%	1.20%	1.36%
6b	1343.36	0.15%	0.04%	0.06%	0.01%	1.15%	0.97%
Prodhon	248413.28	0.95%	-0.31%	-0.55%	-0.36%	4.65%	3.58%
Nguyen	164492.82	0.61%	0.05%	0.28%	0.19%	1.13%	1.38%
Avg.	46703.44	0.40%	-0.03%	0.10%	0.04%	1.15%	1.79%

Set	base	no restart	no 2opt	no 2opt*	no relocate	no swap	no swap 2
2	578.27	0.08%	0.04%	0.10%	0.00%	0.00%	0.00%
3	641.44	0.13%	0.05%	0.09%	0.06%	0.08%	0.18%
4a	1420.05	0.11%	0.05%	0.03%	-0.01%	0.01%	-0.01%
4b	1397.21	0.10%	0.03%	0.27%	0.04%	0.02%	0.03%
5	1132.02	0.06%	0.31%	0.53%	0.47%	0.29%	0.09%
6a	912.51	0.11%	0.02%	0.15%	0.24%	-0.03%	0.02%
6b	1343.36	0.07%	0.05%	0.17%	0.12%	0.05%	-0.01%
Prodhon	248413.28	0.43%	-0.29%	0.30%	0.56%	-0.51%	0.00%
Nguyen	164492.82	-0.08%	-0.20%	0.21%	0.40%	0.22%	0.03%
Avg.	46703.44	0.11%	0.01%	0.21%	0.21%	0.01%	0.04%

## 7 Conclusion

We presented a very simple and fast LNS heuristic for the 2EVRP and 2ELRPD. LNS-2E makes use of one repair operator and only a few destroy operators. The proposed method finds solutions of higher quality than existing algorithms, while being fast and conceptually simpler. The impact of various parameters and design choices was highlighted. Meta-calibration techniques were used to set the parameters to good values, which were subsequently verified during sensitivity analyses. Different techniques were attempted to open or close satellites, and thus explore various combinations of design choices. At the end, a simple randomised approach for fixing satellites, assorted with a minimum number of iterations without change of this decision led to good and robust results on a wide range of benchmark instances. LNS-2E was able to improve 18 best known solutions on the 49 2EVRP instances for which no proven optimal solution exists so far. Having resolved the inconsistencies on the different sets of benchmark instances used in literature paves the way for future research on this topic, which will focus on solving rich city logistics problems, and shifting the cost structure to a more realistic scenario. It will be interesting to examine the implications of using higher operating costs for larger trucks than for smaller city freighters, approaching more realistic and real-life transportation problems.

## Online Resources

All necessary data can be found in the online section at <https://www.univie.ac.at/prolog/research/TwoEVRP>. All instances have been transformed into a uniform format and can be downloaded. Set 4 instances used to have negative and real x/y coordinates (with a maximum of two positions after decimal space). We

added a fixed factor to shift them only to be positive and multiplied them by 100 to be able to use positive integers. This does not change the solution, but note that the objective should be adjusted by a factor 100. We also provide detailed results on the new found best known solutions, both in human readable text and a graphical representation.

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## 8 Appendix

### 8.1 Nomenclature of Set 3 instances

Table 13: Set 3 instances with 50 customers: identical except for depot coordinates

Set 3b <sup>1</sup>		Set 3c <sup>2</sup>	
depot at (0,0)		depot at (30,40)	
E-n51-k5-	12-18	E-n51-k5-	13-19
	12-41		13-42
	12-43		13-44
	39-41		40-42
	40-41		41-42
	40-43		41-44

<sup>1</sup> included in Hemmelmayr (2013)

<sup>2</sup> included in Beasley (2014)

Hemmelmayr et al. (2012) report a best solution with total costs of 690.59 for instance E-n51-k5-s**12-18**, which corresponds exactly to the solution found by our algorithm. On the other hand Jepsen et al. (2012) report an objective value of 560.73 for an instance of that name. This corresponds exactly to the objective value found by our algorithm for the instance E-n51-k5-s**13-19** from Beasley (2014). So we assume Jepsen et al. (2012) used the same instances as are provided from Beasley (2014), but the IDs of satellites in the names have to be increased by one. Both versions exist in literature, as we described in Section 6.1. The only difference between Sets 3b and 3c is the location of the depot. Table 13 shows, which instances correspond to each other, apart from depot coordinates.

### 8.2 Definition of boxplots in Figure 2

The two hinges are versions of the first ( $Q_1$ ) and third quartile ( $Q_3$ ), i.e., close to quantile  $(x, c(1,3)/4)$ . The hinges equal the quartiles for odd  $n$  (where  $n \leftarrow \text{length}(x)$ ) and differ for even  $n$ . Whereas the quartiles only equal observations for  $n = (1 \bmod 4)$ , the hinges do so additionally for  $n = (2 \bmod 4)$ , and are in the middle of two observations otherwise. They are based on asymptotic normality of the median and roughly equal sample sizes for the two medians being compared, and are said to be rather insensitive to the underlying distributions of the samples. The notches extend to  $\pm 1.58 \frac{IQR}{\sqrt{n}}$ , where Interquartile Range  $IQR = Q_3 - Q_1$ . This gives roughly



a 95% confidence interval for the difference in two medians. (The R Foundation for Statistical Computing, 2014)

### 8.3 2ELRPSD instances

The instances for the 2ELRPSD were downloaded at the homepage<sup>3</sup> of Caroline Prodhon. If the description file is renamed to “.doc” it can be opened in human readable format by Word. We found some small typos in the description: At the end of the Prodhon files first the fixed cost of a second level vehicle (F2) is given, then the fixed cost of a truck on first level (F1).

The description of the cost structure also seems to contain an error: According to the description, second level vehicles operate at higher cost per distance than first level trucks, which should be the other way round, obviously. This is also the way we treated the instances, and we believe previous authors did, too. The cost linking point A to point B was calculated as shown in Table 14. Please note that first level vehicles do not operate at exactly twice the Euclidean distance, due to the use of the ceil function *after* multiplication by factor 2.

Table 14: Distance Matrix Calculation for the 2ELRPSD

	Instance set	
	Prodhon	Nguyen
first level	$\lceil \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} * 100 * 2 \rceil$	$\lceil \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} * 10 * 2 \rceil$
second level	$\lceil \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} * 100 \rceil$	$\lceil \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} * 10 \rceil$

In the instance file 200-10-3b, the capacity of first level vehicles is missing, and thus we used 5000 (as this is the value used for all the other Prodhon instance files). Some customers in the 2ELRPSD instances have a demand of 0. This case was not explicitly dealt with in the papers: our algorithm still plans an itinerary for a city freighter which will visit the customer, but does not deliver any quantity of goods.

## Acronyms

**2EVRP** two-echelon vehicle routing problem  
**2ELRP** two-echelon location routing problem  
**2ELRPSD** two-echelon location routing problem with single depot  
**ALNS** adaptive large neighbourhood search  
**CMA-ES** covariance matrix adaptation evolution strategy  
**CVRP** capacitated vehicle routing problem  
**LNS** large neighbourhood search  
**MDVRP** multi-depot vehicle routing problem  
**SDVRP** split delivery vehicle routing problem  
**VNS** variable neighbourhood search  
**VRP** vehicle routing problem

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